The Political Economy of Labor Policy

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Abstract

This article develops a theory that explains the political origins of size-contingent Employment Protection Legislation (EPL), which imposes stricter rules on firms whose number of employees exceeds a certain threshold. In the model, citizens are heterogeneous in wealth and make an occupational choice that determines their voting preferences for EPL. The equilibrium policy only protects workers in larger firms, regardless of the government's primary concern for either workers or entrepreneurs. Firms strategically adjust their labor in response to a size-contingent policy, resulting in welfare distortions. Welfare distortions can be eliminated by properly regulating independent negotiations between workers and entrepreneurs.

Keywords: EPL, S-shaped EPL, interest groups.

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1 Introduction

Employment Protection legislation (EPL) is a set of rules that govern the termination of job contracts. Every country has established a different group of regulations, such as severance payments, reinstatement possibilities, and notification procedures. The primary motivation of EPL is similar in all countries: to shield workers from unfair dismissal. Several policy institutions such as the OECD and the IMF advocate for a reduction of these rigidities as a cure for the high unemployment experienced by regions with highly regulated labor markets, such as Europe. Nevertheless, such reforms have been hard to implement due to considerable political opposition (Saint-Paul, 2002). Possibly as a way to address these challenges, many countries have implemented labor rules that apply differentially according to firm size (size-contingent EPL). However, such regulations are not innocuous: they create a wedge between firms' wages, employment stability, and growth possibilities (Schivardi and Torrini, 2008; Leonardi and Pica, 2013).

In most countries, size-contingent EPL typically takes an S shape, with stricter EPL applying only to firms with the number of employees higher than a certain threshold. For instance, in France, the labor law makes a set of special provisions for firms that have 50 employees or more (Gourio and Roys, 2014). In particular, firms with more than 50 employees must follow a complex redundancy plan in case of collective dismissals. Another example is Italy, where in case of unjustified dismissal firms with more than 15 employees must pay higher damage costs and reinstate the dismissed employee. In the last five decades, S-shaped EPL has been adopted by countries with very different institutional backgrounds and by governments with political positions ranging from left to right (see section 2). This is remarkable, because this regulation is not fully consistent with either ideology. Indeed, S-shaped EPL leaves workers in smaller firms unprotected while imposing higher costs on larger firms. Furthermore, the aggregate costs of EPL are estimated to be rather high, around 3.5% of GDP (Garicano et al., 2016). But if EPL is so costly, why it exists and why it takes an S shape in many countries?

To address these questions, this article builds a political and economic theory that endogeneizes and explains the emergence of S-shaped EPL. In my model, citizens are heterogeneous in wealth and choose to become workers or to start a firm and become entrepreneurs. Workers choose how much labor to supply in response to the equilibrium wage. They are randomly matched to firms, thus they all face the same ex-ante expected utility.[1] Firms are heterogeneous as their investment and labor are limited by endogenous credit constraints that depend on wealth and the strength of EPL. Agents define their voting preferences for EPL by anticipating its ef-

[1] Even though my model does not incorporate a matching technology between workers and entrepreneurs, the equilibrium probability of a worker being matched to a firm with a certain strength of EPL depends on the economy-wide design of regulations. While not explicitly stated, the macro literature studying S-shaped EPL makes a similar assumption regarding how individual workers are matched to different firms (e.g Garicano et al., 2016).

fects on the endogenous variables that determine their occupation-specific decisions. Thus, in my model, wealth heterogeneity and occupational choice give rise to endogenous political preferences for EPL.

The equilibrium policy is determined through probabilistic voting (Lindbeck and Weibull, 1987). The voting model is an application of Persson and Tabellini (2000) to a setting with heterogeneous agents and endogenous political preferences. Initially, workers in all firms are poorly protected against dismissal, so EPL is said to be weak or almost nonexistent. Two political candidates propose an EPL design after making a binary decision for each firm: whether to keep the initially low strength of worker protection or to apply a stronger EPL.[2] Thus, the proposed EPL can be potentially size-contingent. The equilibrium policy maximizes the politically-weighted welfare of workers and entrepreneurs.[3] The weights depend on a parameter measuring the political orientation of the government, either more *pro-worker* or *pro-business*. I characterize the shape of the equilibrium EPL as a function of the government's political orientation. I then study how it depends on whether the wage responds to EPL (*flexible* wages) or not (*sticky* wages).

I start with a baseline model where firm size is defined in terms of assets. Politicians observe the assets' distribution and can choose to apply regulations contingent on assets (*asset-based policy*). The winning candidate can enact and enforce the proposed policy. Thus, I initially rule out strategic behavior of firms, that is, firms cannot adjust or underreport their size in response to regulations. This simplifies the characterization of the equilibrium policy and allows me to derive the main insights of the model. Then, I study a more realistic setting where firm size is defined in terms of labor, and thus, politicians can implement an EPL contingent on labor (*labor-based policy*). In this case, firms strategically adjust their size in response to EPL, resulting in welfare distortions.[4] I show that the qualitative properties of the equilibrium policy remain unchanged. Finally, I study the equilibrium policy that arises from independent negotiations between workers and entrepreneurs. Under certain conditions, the government can eliminate the welfare distortions induced by strategic behavior by properly regulating the bargaining power of workers and entrepreneurs.

The main result is that, when wages are flexible, the equilibrium EPL is S-shaped regardless of the political orientation of the government. That is, there exists an equilibrium size threshold

[2] This is without loss of generality. The results still hold if the economy initially faces strong EPL and candidate governments can decide to apply weaker EPL.

[3] A well-known feature of probabilistic voting models is that in equilibrium, both candidates choose the same platform that maximizes a political objective function which is a weighted average of agents' welfare (in my case, workers and entrepreneurs). In my model, the political weights depend on the wealth distribution and a parameter governing the workers' responsiveness to EPL relative to entrepreneurs, which is my measure of the government's political orientation.

[4] More specifically, a firm can legally avoid being hit by EPL by choosing to hire an amount of labor that is just below the size threshold above which EPL becomes stricter. In many cases, this strategy implies that firms hire an amount of labor that is suboptimal given their investment level.

above which stricter EPL applies. This implies that even when the government cares only about workers, it keeps those in smaller firms unprotected. Conversely, even when the government cares exclusively about entrepreneurs, it subjects larger firms to stricter EPL. More pro-worker governments choose a lower size threshold. These results are consistent with the empirical evidence presented in section 2 and with the findings of Botero et al. (2004) that the left is associated with a more protective EPL.

To establish this result, I start by showing that a flat increase of EPL is neutral, i.e. has no impact on the real economy. Improving EPL in all firms increases the expected labor payment to workers. In response to this increase, workers supply more labor, while entrepreneurs demand less labor leading to a reduction of the equilibrium wage. In equilibrium, the decrease in wage counteracts the initial increase in labor payments. Thus, a homogeneous increase in EPL has no impact on welfare. Can a size-contingent policy improve the political welfare? This article shows that the answer is yes. Moreover, it turns out that such a policy is S-shaped regardless of the political orientation of the government.

The intuition for this result comes from the impact of an S-shaped EPL on the labor market and across different groups of workers and entrepreneurs.[5] First, consider a pro-business government, that cares substantially more about entrepreneurs than workers. Establishing more stringent EPL only on larger firms increases labor market competition, thus reducing the equilibrium wage. Smaller firms substantially benefit from lower wages, while larger firms can more easily absorb stricter EPL due to their easier access to credit. Thus, a pro-business government views an S-shaped EPL as a way to cross-subsidize small firms at a relatively low cost for larger firms. The political motivation for a pro-business government to adopt an S shape EPL can be summarized as follows: *"regulate large businesses to foster small businesses growth"*.

Secondly, consider a pro-worker government. In principle, it would like to provide protection to all workers. However, stricter EPL in smaller firms reduces their already limited access to credit, which discourages investment and hiring. Thus, despite the fact that EPL increases expected labor payments, it significantly decreases employment in the small-scale sector. As a result, the welfare of the group of workers in smaller firms decreases with EPL. Therefore, even though a pro-worker government aims to protect all workers, it chooses to implement softer labor regulations in smaller firms. The core principle of a pro-worker government is summarized as *"do not regulate small businesses to protect their workers"*.

The preceding arguments assume that wages are flexible and adjust to changes in EPL. I show that, when wages are sticky, an S shape EPL is only implemented by more pro-worker govern-

[5]The aggregate welfare of workers in the political objective function can be also written as the sum of the welfare of the group of workers matched to each firm. Thus, it is equivalent to study the solution to the politicians' problem using either one of the two measures. I opt for the latter since it allows for a more insightful interpretation of the results.

ments; otherwise, EPL does not appear in equilibrium. Thus, an S-shaped EPL is more likely to arise in countries where wages are flexible.

In section 6, I study two extensions of the baseline model. First, I consider a more realistic environment where firm size is defined in terms of labor, and thus, politicians can choose to implement a labor-based policy. In this case, firms strategically choose how much labor to hire. Under an S-shaped EPL, a group of firms legally avoid being hit by EPL by hiring an amount of labor just below the size threshold above which EPL becomes stricter.[6] I show that the politicians' problem can be mapped into a problem in which they choose an asset threshold to maximize the labor-based welfare. Thus, the properties of the equilibrium policy can be understood through the lens of the baseline model where size is defined by assets. As a result, the equilibrium EPL remains S-shaped regardless of the political orientation of the government. However, strategic behavior implies that the labor-based welfare is lower than the asset-based welfare. Can politicians use an alternative mechanism to achieve the maximum asset-based welfare (i.e. that survives strategic behavior)?

To address this final question, I study the equilibrium EPL that arises from independent negotiations between groups of workers (unions) and entrepreneurs. Under some conditions, the government can attain the maximum asset-based welfare by using a single-dimensional policy instrument: unions' bargaining power. The explanation for this result comes from the fact that in equilibrium there are no unions in smaller firms. The groups of workers in the small-scale sector anticipate that their firms would seriously struggle to accommodate stricter EPL, negatively impacting their welfare. Thus, workers in smaller firms are aligned with their entrepreneurs in keeping weak EPL. As a result, the government chooses the unions' bargaining power to control the outcome of negotiations in larger firms. The main takeaway is that the government can eliminate the distortions caused by strategic behavior by properly allocating the bargaining power between workers and entrepreneurs.

This paper adds to a vast literature on the political economy of EPL. Saint-Paul (2000) provides a review of the early work on this topic (see also Saint-Paul, 2002). One strand of this literature rationalizes the existence of two-tier systems, where groups of workers *within* a firm coexist under flexible and rigid EPL. These papers build on efficiency wage models along the lines of Shapiro and Stiglitz (1984) (e.g. Saint-Paul, 1996). Much less work has been done to understand size-contingent EPL, which creates a wedge *between* groups of workers and firms. Boeri and Jimeno (2005) took a first step in this direction by showing that if monitoring effectiveness is decreasing in firm size, then stricter EPL can only be accepted in large units. As far as I know, this

[6] As evidence of such strategic behavior, Gourio and Roys (2014) and Garicano et al. (2016) show that the firm size distribution is distorted in France: few firms have exactly 50 employees, while a large number of firms have 49 employees.

paper is the first to develop a theory of endogenous policy choice that rationalizes the emergence of S-shaped EPL across countries.

The macro literature studying size-contingent policies has relied on different extended versions of Lucas (1978) model to estimate the welfare costs of such regulations (Guner et al., 2008; Restuccia and Rogerson, 2008; Garicano et al., 2016; Gourio and Roys, 2014). All these papers take size-contingent regulations as exogenously given. I add to this literature by studying the political origin of size-contingent EPL. The distinctive feature of my model is that the extent to which a firm adapts to EPL depends on its access to credit which is endogenously given by its assets.[7] This interaction between EPL and financial frictions is not present in the aforementioned models and is what gives rise in equilibrium to an S shape EPL.

Finally, my framework relates to the classical models on endogenous credit constraints and occupational choice (Evans and Jovanovic, 1989; Holmstrom and Tirole, 1997). My model is based on the framework developed by Fischer and Huerta (2021). I adapt their setting to allow for firm-specific EPL and a political process that defines the shape of EPL. As shown in section D.6 in the Appendix, my model can be adapted to accommodate other types of size-contingent regulations that are widespread worldwide, such as special tax treatments, credit subsidies, and restrictions on the expansion of businesses. The study of the political economy of these regulations is left for future work.

The paper is organized as follows. Section 2 presents motivating evidence. Section 3 introduces the model. Section 4 describes the conflicts of interest for EPL. Section 5 characterizes the equilibrium policy. Section 6 presents the extensions. Section 7 concludes.

[7] The model captures the recent empirical findings of the literature on labor and finance that EPL distorts firms' decisions by crowding out external finance (Simintzi et al., 2015; Serfling, 2016), discouraging investment (Bai et al., 2020), and reducing employment (Michaels et al., 2019). In my model, this adjustment is greater in smaller firms for which credit constraints get significantly tighter after an improvement of EPL.

2 Motivating Evidence

Following the definition of the OECD (1999), the indicators of employment protection legislation (EPL) evaluate the strength of regulations on the dismissal and hiring of workers. They include both individual and collective dismissal, which are the regulations studied in this paper. Importantly, the focus of this paper is on the extent of EPL, but not on the intensity of regulations. Thus, the data presented is about the coverage of EPL across countries.

Figures 1 and 2 serve as motivation for this paper.[8] The figures plot the firm size threshold (number of workers) at which dismissal regulations become stricter across different countries. The x-axis corresponds to the year in which the size threshold was defined or changed in a given country. The y-axis represents the size threshold from which EPL becomes stricter. The left-hand side panel corresponds to instances in which the size threshold was enacted by a left-wing government (in red), while the right-hand side shows the years in which the regulation was defined by a right-wing government (in blue).[9] The box plots represent the 95% confidence interval around the mean. The top and bottom horizontal lines are the 95th and 5th percentiles, respectively.

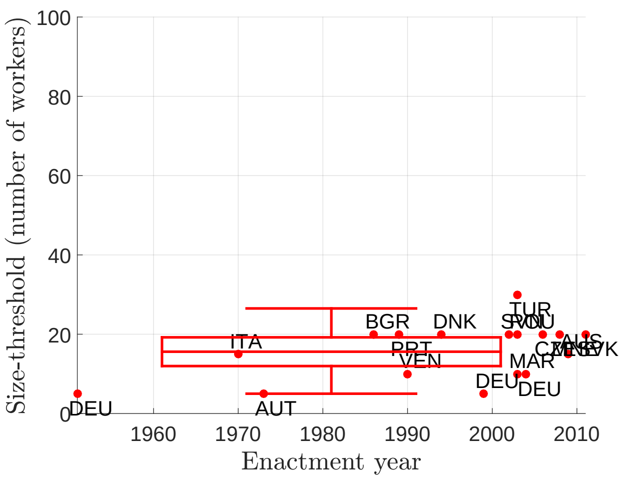

Figure 1: Size threshold, left-wing.

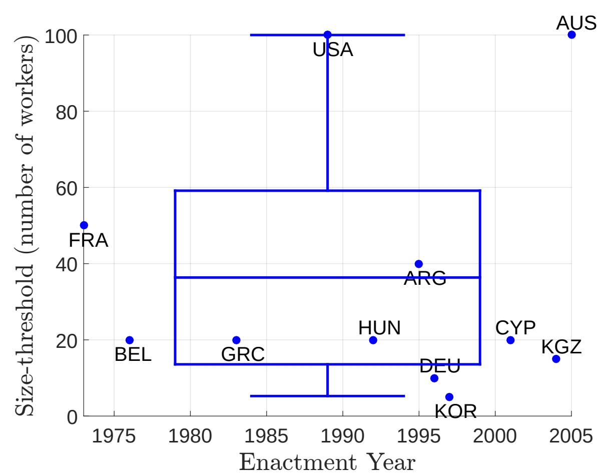

Figure 2: Size threshold, right-wing.

The figures provide three insights regarding EPL. First, and S-shaped EPL has been implemented in many countries and the size threshold from which EPL becomes stricter varies significantly across countries. Secondly, once the size threshold is defined, it remains fixed over time

[8]Source: data collected from different sources, including countries' Labor Codes, the International Labor Organization (ILO) and studies regarding EPL reforms in different countries. Left and right-wing governments are defined on the basis of the political orientation of the executive as measured by the World Bank Database of Political Institutions (WDPI), and defined in Beck et al. (2001). Section C in the Appendix provides more details of data construction.

[9]There are only two instances in which an S shape EPL was adopted by a center government: in 1960, Italy and in 2007, Finland.

for almost all countries.[10] Finally, the average size threshold is lower if it is enacted by a left-wing government, than if it is enacted by a right one.[11]

These facts beg the questions, if left-wing governments supposedly want to protect workers, why do they keep those in smaller firms unprotected? Conversely, if right-wing governments want to protect businesses, why do they impose stricter EPL on larger firms? This paper provides a political economy explanation to these questions.

These facts also guide the model. Since size thresholds remain relatively fixed over time, I study a one-time labor reform. Initially, workers in all firms are poorly protected against dismissal, so EPL is said to be weak. Two political candidates propose an EPL design after making a binary decision for each firm: whether to keep the initially weak EPL or to increase EPL to a certain level. Candidates respond to a parameter measuring their political orientation, either pro-worker (left-wing) or pro-business (right-wing). Citizens vote for their preferred candidate to define the winning regulation. Section 3.4 provides more details.

Table 1 shows how the adoption of S-shaped EPL is distributed across regions and over time. It also presents the number of observations by the political orientation of the executive in the enactment year and by countries' legal origins. Overall, S-shaped EPL has been adopted across several regions and by countries with very different institutional and political backgrounds.

Table 1: Data description

Years	N obs.	Region	N countries	Pol. orientation	N obs.	Legal origins	N countries
1950-1980	6	North America	1	Left	17	French	9
1981-1990	5	South America	2	Center	2	English	3
1991-2000	7	Oceania	1	Right	11	German	3
2001-2011	13	Northern Europe	2			Socialist	8
		Southern Europe	6			Scandinavian	2
		Western Europe	4				
		Eastern Europe	5				
		East Asia	1				
		Western Asia	1				
		Central Asia	1				
		North Africa	1				

[10]There are some exceptions. For instance, Germany has changed the size threshold three times since it was enacted. Also Australia changed its size threshold once.

[11]The average size threshold for left-wing governments is lower than the average threshold for right-wing ones with a 95% level of confidence.

3 The Model

This section outlines the baseline model which builds on Fischer and Huerta (2021, FH hereafter). I develop a tractable framework for the study of the design of Employment Protection Legislation (EPL). Citizens are heterogeneous in wealth and decide between becoming workers or entrepreneurs. Occupational choice determines political interests. Citizens vote according to these interests to determine regulations. The equilibrium EPL is defined through a probabilistic voting mechanism à la Persson and Tabellini (2000). Electoral competition takes place between two political candidates.

In this baseline model, firm size is defined in terms of assets. The political candidates observe the assets' distribution and may choose to apply regulations contingent on assets. Additionally, the winning candidate can enact and enforce the chosen policy. Thus, I initially rule out strategic behavior of firms, i.e. firms cannot adjust or underreport their size in response to EPL. In section 6, I study a more realistic setting in which size is defined in terms of labor, and thus, firms can strategically adjust their size in response to EPL.

3.1 Timeline

Consider a three periods one-good open economy. Figure 3 illustrates the timeline. In what follows I describe the events of each period.

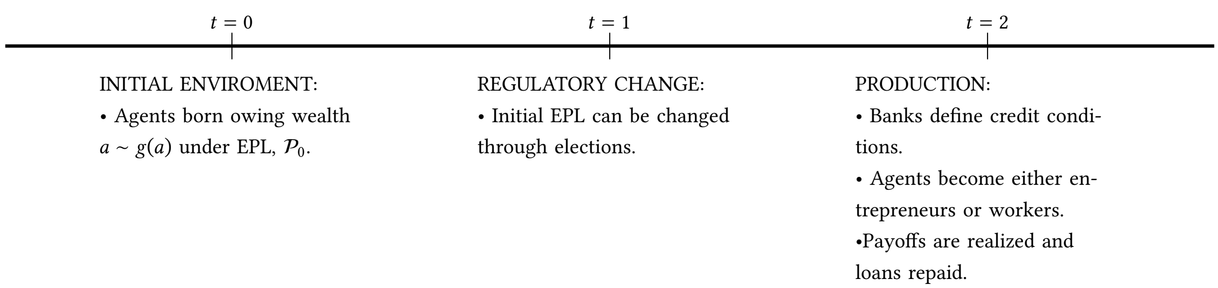

Figure 3: Timeline.

3.1.1 t=0

At $t = 0$, a continuum of risk neutral agents are born differentiated by wealth a. The cumulative wealth distribution $G(a)$ has support in $[0, a_M]$ and continuous density $g(a)$. Agents have access to a Cobb-Douglas production technology given by $f(k, l) = k^\alpha l^\beta, \alpha + \beta < 1$. They are price-takers in the labor and credit markets. The price of the single good is normalized to one. Initially, the strength of EPL is given by $\mathcal{P}_0 = (\varphi_0, \theta_0)$, where $\varphi_0 \in [0, 1]$ measures the strictness of individual dismissal regulations and $\theta_0 \in [0, 1]$ the strength of collective dismissal laws. More details about the working of labor regulations are provided below.

3.1.2 t=1

At $t = 1$, citizens vote to change regulations. The political candidates can increase the strength of individual dismissal regulations from φ_0 to $\varphi_1 = \varphi_0 + \Delta$ and collective dismissal regulations from θ_0 to $\theta_1 = \theta_0 + \Delta$, with $\Delta > 0$. Thus, candidates make a binary decision for each firm: whether to the keep initially weak EPL or to apply stricter EPL.[12] The resulting labor policy is denoted by the function $\mathcal{P}$, which maps firm's assets to a specific strength of EPL, i.e. $\mathcal{P}(a) = (\varphi, \theta)$. The equilibrium EPL is then given by $\mathcal{P} : [0, a_M] \to \Theta$, where $\Theta \equiv \{(\varphi_0, \theta_0), (\varphi_1, \theta_0), (\varphi_0, \theta_1), (\varphi_1, \theta_1)\}$ is the set of feasible policies that can be implemented at each firm.

3.1.3 t=2

At $t = 2$, the economy operates in accordance with the chosen policy, $\mathcal{P}$. The single period is divided into four stages as illustrated by figure 4.

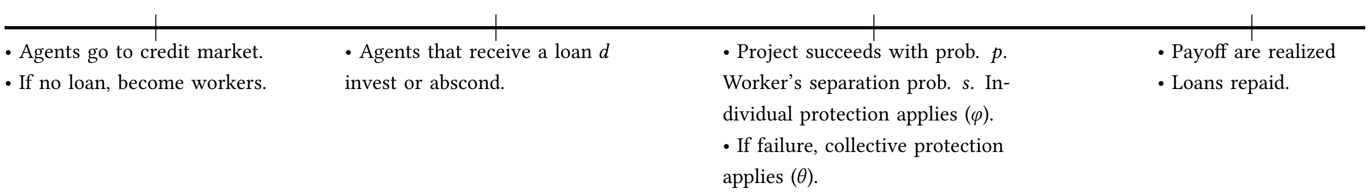

Figure 4: Timing at $t = 2$.

There is a competitive banking system that has unlimited access to funds from abroad at the international interest rate, ρ. Banks provide credit to entrepreneurs while facing a moral hazard problem: investment decisions are non contractible and banks are imperfectly protected against malicious default. In consequence, banks constraint access to credit. As detailed in section 3.3, given the labor policy $\mathcal{P}$, banks set a minimum wealth required to obtain a loan, $\underline{a} \equiv \underline{a}(\mathcal{P}) > 0$ and establish debt limits, $d \equiv d(a|\mathcal{P})$. Excluded agents may become workers ($a < \underline{a}$), the rest can become entrepreneurs ($a \geq \underline{a}$).

Agents receiving a loan ($a \geq \underline{a}$) have two options. First, they can invest their capital in a firm to produce output and become entrepreneurs. Secondly, they may decide to commit *ex-ante* fraud and abscond with the loan to finance private consumption. In this case, only a fraction $1 - \phi$ is recovered by the legal system. Thus, $1 - \phi$ is the loan recovery rate.[13] On the other hand, agents

[12] This is without loss of generality. The results still hold if the firms initially face strong EPL and political candidates can decide to apply weaker EPL.

[13] Fischer et al. (2019) build a model with a similar financial structure where collateral laws are represented by a more general functional form. The results of the model remain unchanged under that more general approach.

excluded from the credit market ($a < \underline{a}$) may become workers at $t = 2$ and supply l_s units of labor. They face a disutility cost of labor given by $\varsigma(l_s) = l_s^\gamma$ with $\gamma > 2$.

There is a fixed cost $F > 0$ of forming a firm, which is paid before payoffs are realized. Firms succeed with probability $p \in (0, 1)$. In that case, they produce output $f(k, (1-s)l)$, where $k = a+d$ is the capital invested by an entrepreneur owning a who asks for a loan d and hires l units of labor. $s \in [0, 1]$ is the job separation probability. When an individual worker is fired, with probability s, entrepreneurs must pay him a fraction $\varphi \in [0, 1]$ of his labor income, given by φwl.[14] Thus, φ is interpreted as a firm-specific measure of firing costs or as the strictness of individual dismissal regulations.

With probability $1 - p$, production fails and bankruptcy procedures take place. As in Balmaceda and Fischer (2009), the legal system recovers only a fraction $\eta \in [0, 1]$ of total invested capital which is distributed among creditors, i.e. banks and workers. First, a fraction $\theta \in [0, 1]$ of labor income wl is paid to workers, the remainder $\eta k - \theta wl$ goes to banks.[15] Hence, θ can be interpreted as the size-specific strength of employees' rights in bankruptcy or more broadly, as the strictness of collective dismissal regulations. Alternatively, it can be understood as a measure of seniority rights of employees of an insolvent firm. That is, when $\theta = 0$ the worker is junior to all creditors, while if $\theta = 1$ she is the most senior of the claimants.

In sum, the strength of EPL in a given firm is represented by the pair (φ, θ), which measures the strictness of individual and collective dismissal regulations, respectively.

3.2 Payoffs

3.2.1 Banks

At the end of period $t = 2$, loans are repaid and outcomes realized. The expected profits of a bank that lends d to an entrepreneur with wealth a, that operates a firm with EPL given by (φ, θ), at the interest rate r is,

$$U^b(a, d, l|\varphi, \theta) = p(1 + r)d + (1 - p)[\eta k - \theta wl] - (1 + \rho)d. \tag{3.1}$$

[14] This can be interpreted as in Saint-Paul (2002), firms are hit by a random shock that destroys the match between workers and entrepreneurs with probability s, in which case the firm pays a firing cost φwl.

[15] Along this paper it is assumed that $\eta k - \theta wl \geq 0$, which simplifies the exposition. If $\eta k - \theta wl < 0$, then all capital recovered goes to workers and banks receive nothing. In that case, the analysis becomes simpler and all results still hold.

3.2.2 Entrepreneurs

The utility of an entrepreneur investing k and hiring l units of labor is,

$$U^e(a, d, l|\varphi, \theta) = p[f(k, (1-s)l) - (1-s)wl - s\varphi wl - (1+r)d] - F. \tag{3.2}$$

3.2.3 Individual workers

The labor utility of an individual worker that supplies l_s units of labor to a firm with EPL (φ, θ) is,[16]

$$\begin{aligned} u^w(l_s|\varphi, \theta) &= p[(1-s)wl_s + s\varphi wl_s] + (1-p)\theta wl_s - \varsigma(l_s), \\ &= \bar{w}(\varphi, \theta) \cdot l_s - \varsigma(l_s), \end{aligned} \tag{3.3}$$

where $\bar{w}(\varphi, \theta) \equiv [p((1-s) + s\varphi) + (1-p)\theta] \cdot w$ is the expected labor payment by unit of labor supplied.[17] Throughout the paper I refer to $\bar{w}$ as the *expected wage*.

As in the macro literature studying size-contingent EPLs (Gourio and Roys, 2014; Garicano et al., 2016), I assume that individual workers are randomly matched to firms of different sizes. That is, there is not a matching mechanism through which individual workers are assigned to firms. Thus, the ex-ante expected utility of individual workers is the same. I denote by $\mathbb{E}u^w(\mathcal{P})$ the expected utility of an individual worker given regulations $\mathcal{P}$.[18]

3.2.4 Group of workers

Finally, define the total utility of workers matched to a firm with labor regulations (φ, θ) that hires l units of labor,

$$U^w(l|\varphi, \theta) = n \cdot u^w \equiv \frac{l}{l_s} \cdot [\bar{w}(\varphi, \theta) \cdot l_s - \varsigma(l_s)] = \bar{w}(\varphi, \theta) \cdot l - \frac{l}{l_s}\varsigma(l_s), \tag{3.4}$$

where $n \equiv l/l_s$ is a measure of the 'number' of workers hired by the firm. Intuitively, U^w represents the utility of the group of workers in a firm hiring l units of labor (or total workers' welfare in a firm).[19]

[16] She also obtains $(1+\rho)a$ from depositing her wealth in the banking system. Thus, total worker's utility is $u^w + (1+\rho)a$.

[17] Observe that this measure depends on the equilibrium wage w, which is a function of economy-wide labor regulations, $\mathcal{P}$.

[18] Note that the expectation comes from the fact that there is some endogenous probability of being matched to a firm with a given strength of EPL. Section A.4 in the Appendix provides an explicit expression for $\mathbb{E}u^w(\mathcal{P})$ when EPL is S-shaped.

[19] Section A.3 in the Appendix shows that U^w is an 'appropriate' measure of workers' utility in a given firm.

The following condition must be satisfied given any labor policy $\mathcal{P}$

$$\mathbb{E}u^w(\mathcal{P}) \cdot G(\underline{a}_0) = \mathbb{E}_G[U^w|\mathcal{P}], \tag{3.5}$$

where $\mathbb{E}u^w(\mathcal{P}) \cdot G(\underline{a}_0)$ is the total worker's welfare and $\mathbb{E}_G[U^w|\mathcal{P}]$ is the weighted sum of the utilities of each group of workers at each firm. Hence, U^w indicates how the total workers' welfare is distributed across firms. As will be clear in section 3.4, the solution to the politician problem can be characterized by using either of the two measures. I opt for using U^w because it allows for a more insightful interpretation of the results.

3.3 Ex-ante competitive equilibrium

This section describes the competitive equilibrium that would arise if the economy operates under the initial EPL given by $\mathcal{P}_0 = \{\varphi_0, \theta_0\}$. The political preferences of the different groups of agents are defined on the basis of this *ex-ante* competitive equilibrium. That is, given $\mathcal{P}_0$ and a, agents understand what their position in society would be and how an improvement of EPL would affect them relative to this initial position. In section 4, I study in detail these political preferences.

3.3.1 Workers' decisions

To find the individual labor supply, l_s each worker maximizes (3.3) to obtain,

$$\varsigma'(l_s) = \bar{w}(\varphi_0, \theta_0) = [p((1-s) + s\varphi_0) + (1-p)]\theta_0] \cdot w. \tag{3.6}$$

Thus, l_s is defined as the level of labor that equalizes the marginal labor benefit $\bar{w}(\varphi_0, \theta_0)$ with the marginal effort cost $\varsigma'(l_s)$.[20]

3.3.2 Banks' decisions

The banking system is assumed to be competitive. Imposing the zero-profits condition in (3.1) gives,

$$1 + r = \frac{1+\rho}{p} - \frac{1}{pd}(1-p)[\eta k - \theta_0 wl], \tag{3.7}$$

where $1 + r$ is the interest rate charged to an entrepreneur that operates a firm with debt d, investment $k = a + d$, and labor l.

[20]Note that individual labor supply does not depend on a.

3.3.3 Entrepreneurs' decisions

Replacing (3.7) in (3.2),

$$U^e(a, d, l|\varphi_0, \theta_0) = pf(k, (1-s)l) + (1-p)\eta k - \bar{w}(\varphi_0, \theta_0)l - (1+\rho)d - F. \tag{3.8}$$

Thus, expected entrepreneur's utility can be rewritten as the expected value of the firm $pf(k, (1-s)l) + (1-p)\eta k$ net of expected labor costs $\bar{w}(\varphi_0, \theta_0)l$, credit costs $(1+\rho)d$, and sunk costs F. The entrepreneur's problem is

$$\max_{d,l} U^e(a, d, l|\varphi_0, \theta_0)$$

$$s.t. \;\; U^e(a, d, l|\varphi_0, \theta_0) \geq u^w(\varphi_0, \theta_0) + (1+\rho)a, \tag{3.9}$$

$$U^e(a, d, l|\varphi_0, \theta_0) \geq \phi k, \tag{3.10}$$

where (3.9) and (3.10) are the participation and incentive compatibility constraints, respectively. Condition (3.9) asks that the agent prefers to form a firm instead of becoming a worker and (3.10) states that the entrepreneur does not have incentives to abscond with the loan. Solving the unconstrained problem leads to the optimal firm size given by capital $k_0^* \equiv k^*(\varphi_0, \theta_0)$ and labor $l_0^* \equiv l^*(\varphi_0, \theta_0)$,

$$pf_k(k_0^*, (1-s)l_0^*) = 1 + r^* \equiv 1 + \rho - (1-p)\eta, \tag{3.11}$$

$$p(1-s)f_l(k_0^*, (1-s)l_0^*) = \bar{w}(\varphi_0, \theta_0). \tag{3.12}$$

Note that (k_0^*, l_0^*) corresponds to the efficient operation scale if loans were not limited by financial frictions. As a consequence of credit market imperfections, only sufficiently rich entrepreneurs will operate efficiently. Section A.1 in the Appendix describes the conditions of the optimal debt contract. The non-absconding condition (3.10) defines two critical wealth thresholds. First, a minimum level of wealth required to obtain a loan, $\underline{a}_0$. Secondly, a minimum wealth, $\overline{a}_0$ to obtain a loan to operate at the efficient scale. Thus, agents with $[\underline{a}_0, \overline{a}_0)$ can obtain a loan which allows them to start a firm, but must operate at an inefficient scale, i.e. they invest $k < k_0^*$.

Additionally, note that restriction (3.9) defines a third critical wealth level, $\hat{a}_0$ from which agents prefer to establish a firm instead of becoming workers. Section A.2 in the Appendix briefly describes the different arrangements that could arise in the model as a function of $\underline{a}_0$ and $\hat{a}_0$. For simplicity, I consider the case in which $\underline{a}_0 > \hat{a}_0$, that is agents excluded from the credit market prefer to become workers instead of forming a firm.[21]

[21] FH show that the features of the model remain qualitatively unchanged in the remaining cases.

3.3.4 Ex-ante equilibrium

Overall, the model sorts agents into four groups: i) workers ($a < \underline{a}_0$), ii) entrepreneurs operating inefficient firms ($a \in [\underline{a}_0, \overline{a}_0)$), iii) entrepreneurs obtaining credit to operate efficiently ($a \in [\overline{a}_0, k^*)$, and iv) entrepreneurs that self-finance an efficient firm ($a \geq k_0^*$). Figure 5 summarizes these features. As shown by equations (A.6) and (A.7) in the Appendix, the optimal decisions of entrepreneurs can be written in terms of wealth, i.e. $d = d(a)$ and $l = l(a)$. Hence, entrepreneurs' and workers' utilities can be simply denoted as $U^e(a|\mathcal{P})$ and $U^w(a|\mathcal{P})$, respectively.

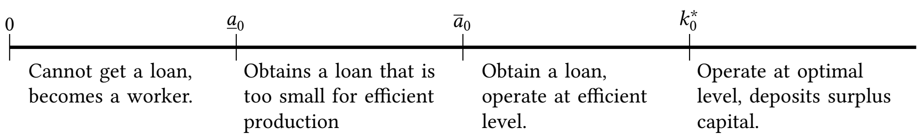

Figure 5: Ex-ante competitive equilibrium.

Finally, the labor market equilibrium wage w arises from

$$l_s \cdot G(\underline{a}_0) = \int_{\underline{a}_0}^{\overline{a}_0} l \, \partial G(a) + l^*(1 - G(\overline{a}_0)), \tag{3.13}$$

where the left-hand side is total labor supply and the right-hand side is labor demand. This condition uniquely defines the equilibrium wage w.

3.4 The problem of politicians

In this section, I start by presenting the problem of politicians. After I have described the problem, I explain how it can be microfounded through a political process.

Consider a politician that chooses a policy design, $\mathcal{P} = (\mathcal{P}^\varphi, \mathcal{P}^\theta)$, where $\mathcal{P}^\varphi$ and $\mathcal{P}^\theta$ denote the individual and collective dismissal regulations, respectively. At $t = 1$, the political candidate makes a binary decision for each firm with assets a: whether to keep weak EPL or to increase the strength of EPL. She can improve individual dismissal regulations from φ_0 to φ_1. In the case of collective regulations, she can increase them from θ_0 to θ_1. Thus, the 'political equilibrium' is given by the policy functions $\mathcal{P}^\varphi : [0, a_M] \to \{\varphi_0, \varphi_1\}$ and $\mathcal{P}^\theta : [0, a_M] \to \{\theta_0, \theta_1\}$, that map firms' assets to their specific strength of EPL. Recall that $\varphi_1 = \varphi_0 + \Delta$ and $\theta_1 = \theta_0 + \Delta$, where $\Delta > 0$.

The candidate responds to its political orientation, which ranges from left-wing (pro-worker) to right-wing (pro-business). The relative importance of workers over entrepreneurs in the politician's problem is measured by $\lambda \in [0, 1]$. Therefore, as λ increases, workers receive a larger weight relative to entrepreneurs. More leftist governments are represented by a larger λ, while right-wing ones by lower values of λ.

The *political objective function* corresponds to the ex-post weighted-welfare denoted by $\bar{U}(\mathcal{P})$. The equilibrium policy arises from maximizing $\bar{U}(\mathcal{P})$ given $\mathcal{P}_0$ and subject to the labor market equilibrium condition,[22]

$$\begin{aligned} &\max_{\mathcal{P}=\{\mathcal{P}(a)\}_0^{a_M}} \{\bar{U}(\mathcal{P}) \equiv \lambda \cdot \mathbb{E}_G[U^w|\mathcal{P}] + (1-\lambda) \cdot \mathbb{E}_G[U^e|\mathcal{P}]\} \\ &s.t. \quad \mathbb{E}_G[l_s|\mathcal{P}] = \mathbb{E}_G[l|\mathcal{P}] \end{aligned} \quad , \tag{3.14}$$

where the constraint corresponds to the analogous of (3.13), but in this case the labor policy is allowed to depend on firm size.[23]

In section D.1 in the Appendix, I provide an explicit microfoundation for this problem. I show that the problem can be rationalized as a probabilistic voting model along the lines of Persson and Tabellini (2000, pp. 52-58). The political weight λ depends on the primitives of the model and on the endogenous mass of workers, $G(\underline{a}_0)$. The electoral competition takes place between two parties that simultaneously announce their electoral platforms, $\mathcal{P}$ to maximize their probability of winning the election.

Note that the expression for $\bar{U}$ is defined for any policy $\mathcal{P}$ as the weighted expected value of workers' and entrepreneurs' welfare. As it stands, the problem is cumbersome. First, there is no restriction on the labor policy design that maximizes $\bar{U}$. In principle, it could exhibit any shape and trying to solve the problem implies checking all possible solutions. Secondly, because $\mathcal{P}$ can have any shape, one should keep track of what agents are subject to a given EPL's regime. This complicates the expression for $\bar{U}(\mathcal{P})$, which takes different forms depending on the shape of $\mathcal{P}$. Moreover, the equilibrium condition must 'clear' the labor supplied and demanded by all subsets of agents subject to a given EPL's regime.

In order to solve the problem, in section 4, I start by studying the agents' political preferences for EPL. Next, in section 5, I show that these endogenous preferences restrict the solution of the politician's problem to the set of functions that satisfy monotonicity at each component.

[22]The dependence on $\mathcal{P}_0$ comes from the fact that the government is deciding whether to increase individual and collective dismissal regulations of each firm from (φ_0, θ_0) to $(\varphi, \theta) \in \{(\varphi_0, \theta_0), (\varphi_1, \theta_0), (\varphi_0, \theta_1), (\varphi_1, \theta_1)\}$. In addition, the individual political preferences for EPL are defined on the basis of the ex-ante equilibrium presented in section 3.3.

[23]Note that the political objective function is equivalent to $\bar{U}(\mathcal{P}) \equiv \lambda \cdot \mathbb{E}u^w(\mathcal{P}) \cdot G(\underline{a}_0) + (1-\lambda) \cdot \mathbb{E}_G[U^e|\mathcal{P}]$, where $\mathbb{E}u^w(\mathcal{P})$ is the expected utility of individual workers under $\mathcal{P}$, which is homogeneous across workers. Recall that: $\mathbb{E}u^w(\mathcal{P}) = \mathbb{E}_G[U^w|\mathcal{P}]$ (equation (3.5) in section 3.2). That is, the aggregate workers' welfare is equal to the sum of the welfare of workers in each firm. It is equivalent to solve the politician's problem using either of the two measures. I opt for using $\mathbb{E}_G[U^w|\mathcal{P}]$ as it allows for a more insightful interpretation of the results.

4 Political Preferences

This section describes the conflicts of interest that arise between the different groups of workers and entrepreneurs for EPL. Given the initial policy, $\mathcal{P}_0$ I analyze the ex-post effect (at $t = 2$) of a marginal increase of EPL (at $t = 1$) on workers' (U^w) and entrepreneurs' utilities (U^e).

First, impose the following assumption on the probability of success of a firm,

Assumption 1 $p > \frac{1}{\eta}\left[\frac{\alpha\phi}{\beta(1-s)^2(1-\alpha-\beta)} - (1+\rho) + \eta\right] \Leftrightarrow 1 + r^* > \frac{\alpha\phi}{\beta(1-s)^2(1-\alpha-\beta)}$.

This is a sufficient condition for propositions 1 and 2 to hold.[24]

Proposition 1 *Consider the initial labor regulation, $\mathcal{P}_0 : [\underline{a}_0, a_M] \to \{\varphi_0, \theta_0\}$, then:*

1. *All entrepreneurs are worse off after a marginal increase of φ or θ.*

2. *This negative effect is strictly decreasing if $a \in [\underline{a}_0, \overline{a}_0)$ and remains constant after $a \geq \overline{a}_0$.*

Proposition 1 shows that the ex-post (at $t = 2$) effect of increasing the strength of EPL is negative for all entrepreneurs. First, raising φ means that firms face higher individual dismissal costs, i.e. higher expected wage, $\bar{w}(\varphi, \theta)$. Therefore, entrepreneurs have less capital to be pledged to banks and more incentives to behave maliciously. Secondly, higher θ implies that less capital is recovered by banks in case of bankruptcy. Thus, banks tighten credit requirements, limiting firms' operations. For smaller firms, this effect is more pronounced as their access to credit is substantially reduced. This leads to significantly lower investment and hiring in the small-scale sector. In contrast, credit capacity of better capitalized firms is less affected. Moreover, many of them have unused debt capacity. Thus, larger firms can more easily adapt to higher labor costs and continue operating at a relatively more efficient scale compared to poorer firms.

To sum up, all entrepreneurial groups oppose a marginal increase of EPL. The strongest opposition to such policies comes from entrepreneurs running smallest firms, while large entrepreneurs are less reluctant to improvements of EPL.

When EPL increases for a non-negligible mass of firms there are also general equilibrium effects that occur due to a change in the equilibrium wage. Section 5.2 explores these effects. The discussion of this section only considers the impact of a marginal increase of EPL on a given firm

[24] This assumption is in general not very restrictive, as the lower bound for p is negative for a large set of 'reasonable' parameters. When it is binding, it does not limit p significantly. For instance, for $\rho = \frac{5}{12}\%, \phi = 15\%, \eta = 70\%, \alpha = 0.25, \beta = 0.6, s = 2.5\%$ it asks that $p > 0.192$.

or on the utility of a given group of workers, that is without taking into account the affects on wages.[25]

Proposition 2 *Consider the initial labor regulation, $\mathcal{P}_0 : [\underline{a}_0, a_M] \to \{\varphi_0, \theta_0\}$ and suppose a marginal increase of φ or θ. Then, there are cutoffs $\tilde{a}_0^{\varphi} \in (\underline{a}_0, \overline{a}_0)$ and $\tilde{a}_0^{\theta} \in (\underline{a}_0, \overline{a}_0)$ given by*

$$\frac{\partial U^w(\tilde{a}_0^x|\mathcal{P}_0)}{\partial x} = 0, \quad x \in \{\varphi, \theta\} \tag{4.1}$$

such that,

1. *Workers' welfare in firms with $a \in [\underline{a}_0, \tilde{a}_0^x)$ decreases.*
2. *Workers' welfare in firms with $a > \tilde{a}_0^x$ increases.*
3. *This marginal effect is strictly increasing in $a \in [\underline{a}_0, \overline{a}_0)$ and remains constant after $a \geq \overline{a}_0$.*

Proposition 2 suggests the existence of interest groups of workers with diverging political preferences for EPL. Strengthening EPL, which supposedly protects workers, has an ambiguous effect on their welfare depending on the firm they are matched to. Two opposing effects determine the direction of the effect of increased EPL: i) higher expected labor payments, but ii) stricter credit constraints which force some firms to shrink and hire less labor.

The welfare of groups of workers in smaller firms ($a \in [\underline{a}_0, \tilde{a}_0^x)$) declines. In some cases firms close down, because the entrepreneur does not obtain financing under the new conditions. SMEs that survive have to shrink. Since they obtain smaller loans, less capital is invested and thus, less labor is hired. This negative effect is particularly pronounced in under-capitalized firms, for which banks severely constraint loans. Hence, workers in the small business sector are made worse off.

On the other hand, an improvement of EPL increases the welfare of workers in larger firms ($a > \tilde{a}_0^x$). Despite the fact that some of these enterprises face tighter credit constraints and hire less labor, this is compensated by the increase in workers' payment in case of dismissal, leading to an increase of workers' welfare. Figure 6 illustrate propositions 1 and 2. It shows the marginal impact of increased EPL on U^e and U^w as a function of firm assets, a. The blue dashed line corresponds to entrepreneurs and the red solid line to workers.

Overall, workers in under-capitalized firms are aligned with small entrepreneurs in opposing to stricter EPL. In contrast, those in larger firms are in favour of stronger EPL and opposed to their

[25] However, note that the proofs of propositions 1 and 2 are more general. They consider the possibility of having an indirect effect through wages ($\frac{dw}{dx}, x \in \{\varphi, \theta\}$), which would occur if a non-negligible mass of firms experienced an increase in EPL. Both propositions hold as long as EPL doesn't improve in all firms. In that case, the net effect on expected wages is zero (see lemma 2 in section 5.2.1).

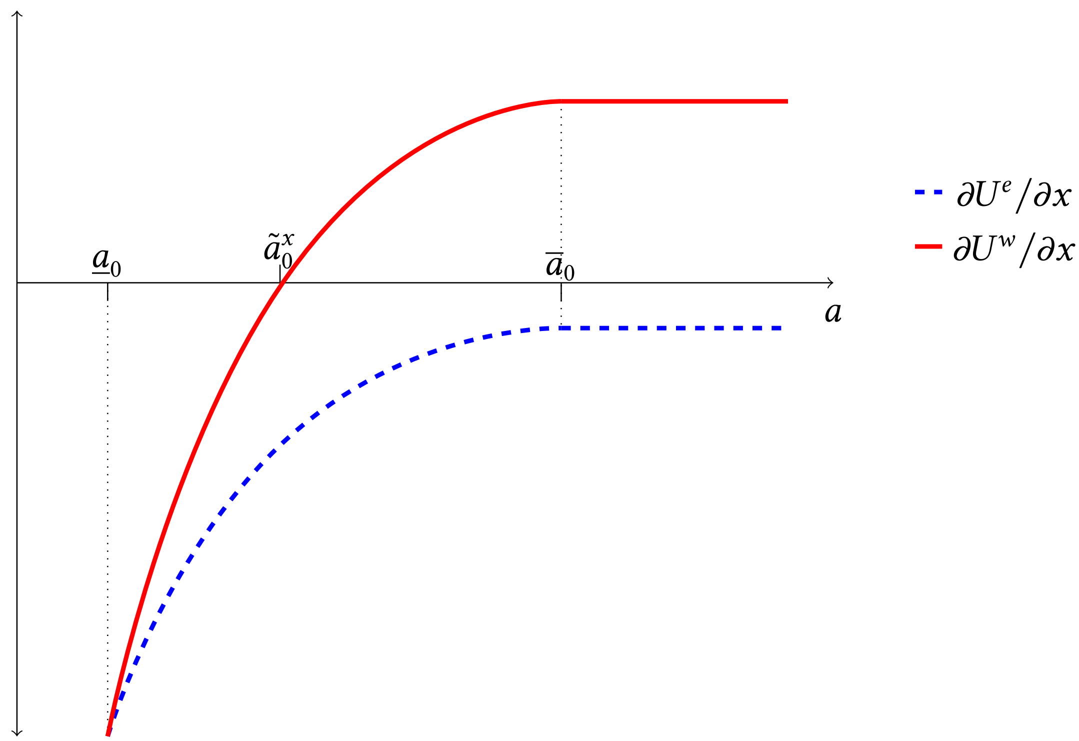

Figure 6: Effects of an increase of $x = \{\varphi, \theta\}$ on entrepreneurs' and workers' utility.

employers' interests. Table 2 summarizes the political preferences of workers and firms across different business sectors.[26]

	Worker	Entrepreneur
Small-medium scale sector; $a \in [\underline{a}_0, \tilde{a}_0^x)$	< 0	$<< 0$
Large scale sector; $a > \tilde{a}_0^x$	> 0	< 0

Table 2: Political preferences towards stricter EPL ($\uparrow x \in \{\varphi, \theta\}$).

[26]'< 0' indicates opposition to EPL, while '> 0' denotes support for EPL. '$<< 0$' stands for strong opposition.

5 Political Equilibrium

This section characterizes the political equilibrium. That is, the labor policy that solves the problem of the politician (3.14) in accordance with her political orientation, λ. I start by showing that the solution to this problem is monotone. Then, in section 5.1, I study the equilibrium policy when wages are sticky. Finally, in section 5.2, I study the political equilibrium under flexible wages.

The next proposition exploits the properties of individual preferences studied in previous section to show that any equilibrium policy must satisfy monotonicity in each component. That is, there are two firm size thresholds, $a^{\varphi} \in [\underline{a}_0, a_M]$ and $a^{\theta} \in [\underline{a}_0, a_M]$, at which individual and collective dismissal protection become more stringent. This result allows me to write $\bar{U}$ more explicitly and makes the politician's problem tractable. Note that this result does not necessarily imply that the equilibrium policy is S-shaped. It restricts the solution of the politician's problem to policies that are either flat or S-shaped.

Let x_i, with $i \in \{0, 1\}$ be defined as,

$$x_i = \begin{cases} \varphi_i & \text{if } x = \varphi, \\ \theta_i & \text{if } x = \theta. \end{cases}$$

Proposition 3 *Any labor regulation policy, $\mathcal{P}$ that solves* (3.14) *satisfies monotonicity at each component,*

$$\mathcal{P}^x(a) \, : \, \mathcal{P}^x(a') \leq \mathcal{P}^x(a'') \quad \forall a' < a'', x \in \{\varphi, \theta\}.$$

Moreover, there are size thresholds $a^{\varphi} \in [\underline{a}_0, a_M]$ and $a^{\theta} \in [\underline{a}_0, a_M]$ such that:

$$\mathcal{P}^x(a) = \begin{cases} x_0 \; if \, a < a^x, \\ x_1 \; if \, a \geq a^x. \end{cases} \tag{5.1}$$

To simplify the exposition, in the rest of the paper I work with the case in which politicians propose a regulatory change in a single dimension. That is, politicians consider increasing either individual or collective dismissal regulation, but not both at the same time. In section D.2 in the Appendix, I study the two-dimensional case, when both individual and collective dismissal regulations are defined through elections. I show that the equilibrium policy remains S-shaped in both dimensions, that is, there are two size thresholds above which each regulation becomes stricter. This is consistent with the kind of labor rules that apply in Austria and France.

Using the result of proposition 3, the politician's problem can be rewritten in terms of the size threshold, a^x as follows,

$$\max_{a^x \in [\underline{a}_0, a_M]} \Bigg\{ \bar{U}(a^x, \lambda) \equiv \lambda \left(\int_{\underline{a}_0}^{a^x} U^w(a|x_0)\partial G + \int_{a^x}^{a_M} U^w(a|x_1)\partial G \right)$$

$$+ (1-\lambda) \left(\int_{\underline{a}_0}^{a^x} U^e(a|x_0)\partial G + \int_{a^x}^{a_M} U^e(a|x_1)\partial G \right) \Bigg\}$$

$$s.t \qquad m^0 \cdot l_s(x_0) = \int_{\underline{a}_0}^{a^x} l(a|x_0)\partial G, \tag{5.2}$$

$$m^1 \cdot l_s(x_1) = \int_{a^x}^{a_M} l(a|x_1)\partial G, \tag{5.3}$$

$$m^0 + m^1 = G(\underline{a}_0), \tag{5.4}$$

where $\bar{U}(a^x, \lambda)$ is the politically-weighted welfare given the size threshold, a^x and the government's political orientation, λ. In the rest of the paper, I refer to $\bar{U}(a^x, \lambda)$ as the *asset-based welfare*. Also, m^0 and m^1 are the endogenous masses of workers that supply $l_s(x_0)$ and $l_s(x_1)$, respectively. The three restrictions of the problem correspond to the labor market equilibrium conditions. The first two equations equalize labor supplied and demanded under the two different EPL regimes, x_0 and x_1. The last condition imposes that the sum of workers under x_0 and x_1 must be equal to the total mass of workers, $G(\underline{a}_0)$.

Note that equations (5.2) to (5.4) form a system of three equations and three unknowns: m^0, m^1 and w. The equilibrium wage w is uniquely defined by these conditions. Adding up (5.2) and (5.3) leads to an aggregate condition similar to the labor market equilibrium condition (3.13),

$$m^0 \cdot l_s(x_0) + m^1 \cdot l_s(x_1) = \int_{\underline{a}_0}^{a^x} l(a|x_0)\partial G + \int_{a^x}^{a_M} l(a|x_1)\partial G, \tag{5.5}$$

Next sections characterize the political equilibrium that arises from solving the politician's problem.

5.1 Political equilibrium with sticky wages

I start by studying the case in which the equilibrium wage is sticky and equal to the value that solves (3.13) under the initial labor policy $\mathcal{P}_0$. Thus, the politicians maximize the asset-based welfare by taking the wage, $w^0 = w(\mathcal{P}_0)$, fixed. This is a useful starting point before analyzing the more complicated case in which the equilibrium wage responds to changes of the size threshold, a^x. Section 5.2 studies this case. This section is divided into two subsections. Subsection 5.1.1 presents the political preferences under sticky wages. Subsection 5.1.2 characterizes the equilibrium policy.

5.1.1 Political preferences with sticky wages

This subsection describes the political preferences for the size threshold above which stricter EPL applies, a^x. Since wages are sticky, agents that are not affected by the regulatory change remain indifferent. In section 5.2, when wages are flexible all agents are affected by a change in regulations, even if they remain subject to the initially weak EPL.

The political preferences can be inferred from propositions 1 and 2 of previous section. Figures 7 and 8 illustrate the change in workers and entrepreneurs utilities as function of the size threshold, a^x. The changes are relative to the utilities they would obtain under the initial labor policy, $\mathcal{P}_0$. All agents are indifferent when they are not affected by the change in regulations, i.e. when their firms' assets are such that $a < a^x$.

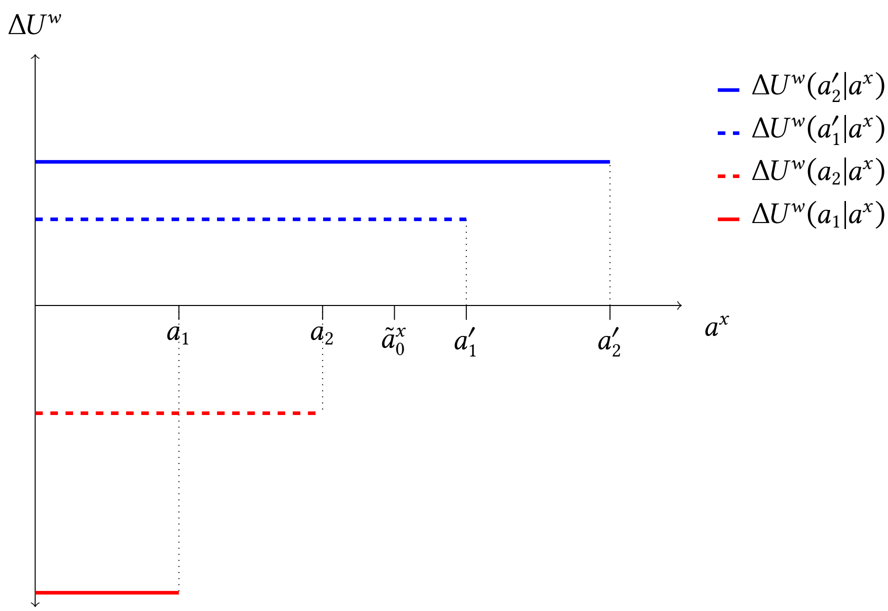

Figure 7: ΔU^w as function of a^x, sticky wage.

The red lines in figure 7 show that groups of workers in firms with assets $a < \tilde{a}_0^x$ are worse off whenever their firms are subject to stricter EPL, i.e. whenever $a \geq a^x$. In contrast, as shown by the blue lines, those workers in firms with $a > \tilde{a}_0^x$ benefit from a change in regulations as long as they receive higher protection, i.e. if $a > a^x$.

The figure also compares the utility losses of workers matched to firms of four different sizes: $a_1 < a_2 < \tilde{a}_0^x$ and $a_2' > a_1' > \tilde{a}_0^x$. The red solid line shows that workers in smaller firms (a_1) suffer more from EPL than those in larger firms (a_2), represented by the red dashed line. On the other

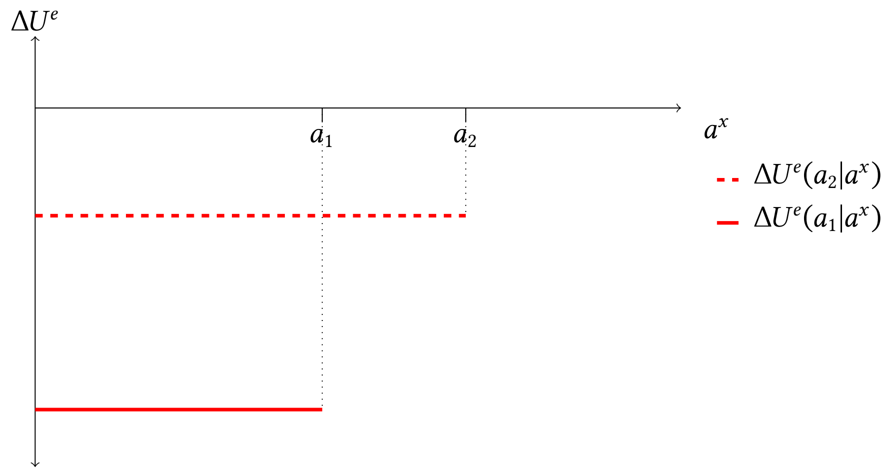

Figure 8: ΔU^e as function of a^x, sticky wage.

hand, those workers matched to larger firms (a_2') gain more from EPL than those in smaller firms (a_1').

Figure 8 depicts entrepreneurs' utilities as a function of a^x. All entrepreneurs are worse off under stricter EPL, i.e. when $a > a^x$. Those running smaller firms (a_1) suffer more from EPL than owners of larger firms (a_2).

What is the shape of the asset-based welfare, $\bar{U}$ that results from aggregating these preferences? Figure 9 illustrates $\bar{U}$ as a function of a^x and λ. The value of $\bar{U}$ at $\mathcal{P}_0$ is normalized to zero in the figure. Thus, if the politician does not implement any regulatory change, i.e. if she sets $a^x = a_M$, then $\bar{U} = 0$. As shown in the figure, the shape of $\bar{U}$ depends on λ.

First, when the politician cares only about workers ($\lambda = 1$), then $\bar{U}$ is single-peaked at $\tilde{a}_0^x$, as shown by the continuous red line in the figure. Therefore, the political equilibrium when $\lambda = 1$ is $a^x = \tilde{a}_0^x$. Secondly, if the politician cares only about entrepreneurs ($\lambda = 0$), then $\bar{U}$ is negative in $[0, a_M]$ and increasing in a^x, since poorer entrepreneurs suffer more from EPL. This is shown by the dashed-blue line. In this case, the politician chooses not to improve EPL, i.e. $a^x = a_M$.

The question that remains is: what is the shape of $\bar{U}$ for $\lambda \in (0, 1)$? This situation is illustrated by the dotted line. Intuitively, for a relatively low λ, the welfare should remain negative for any size threshold, thus $a^x = a_M$. Conversely, for a relatively high λ, $\bar{U}$ should still have a single peak at some asset threshold that gives $\bar{U} > 0$. For intermediate values of λ, the function may have more than one peak depending on the shape of the wealth distribution. Moreover, the peak may give a negative value for $\bar{U}$. Next subsection defines the set of λ's for which a political equilibrium can be characterized.

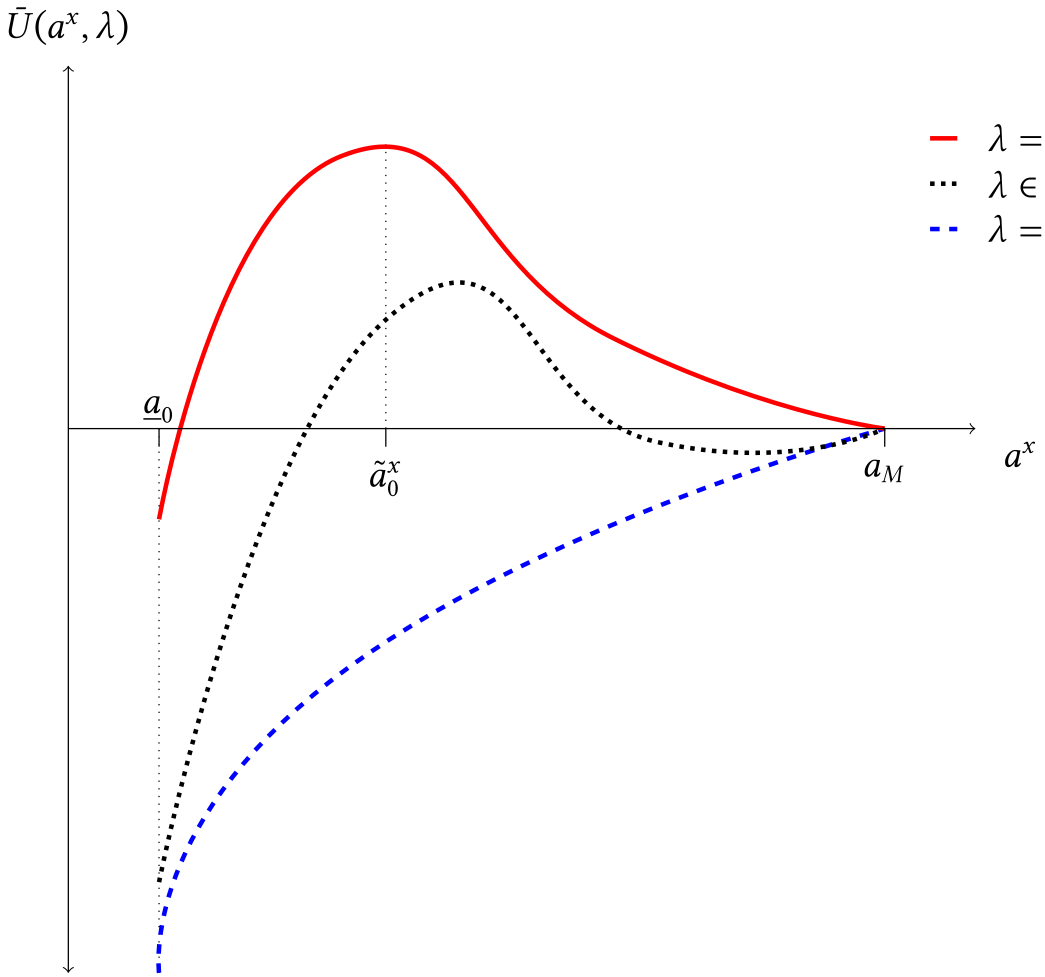

Figure 9: Asset-based welfare ($\bar{U}$) as function of λ and a^x, sticky wage.

5.1.2 Equilibrium labor policy with sticky wages

The following proposition characterizes the political equilibrium, given by the size threshold, a^x_{pe} that maximizes the asset-based welfare, $\bar{U}$.

Proposition 4 *The equilibrium size threshold, a^x_{pe} under sticky wages is as follows:*

1. *If $\lambda \leq \frac{1}{2+1/(\gamma-2)}$, then $a^x_{pe} = a_M$.*
2. *If $\lambda > \frac{1}{2-1/\gamma}$, then $a^x_{pe} \in [\tilde{a}^x_0, \overline{a}_0)$ satisfies,*

$$\lambda \frac{\partial U^w(a^x_{pe}|x_0)}{\partial x} = -(1-\lambda)\frac{\partial U^e(a^x_{pe}|x_0)}{\partial x}. \tag{5.6}$$

In particular, if $\lambda = 1$, then $a^x_{pe} = \tilde{a}^x_0$ and $a^x_{pe} > \tilde{a}^x_0$ if $\lambda < 1$.

The interpretation of proposition 4 is as follows: when the politician cares relatively more about entrepreneurs than workers' wellbeing ($\lambda \leq \frac{1}{2+1/(\gamma-2)}$) it is better not to change regulations and keep a low level of EPL across the board. As discussed in previous section, improving EPL only harms entrepreneurs. Since banks curtail credit after an improvement of EPL, less capital is invested and less labor is hired, which negatively affects output. Therefore, it is not worthy from the point of view of a right-wing politician to sacrifice production in exchange of increasing workers' welfare.

On the contrary, when the politician cares relatively more about workers ($\lambda > \frac{1}{2-1/\gamma}$) the equilibrium EPL is S-shaped. That is, a regulatory scheme in which small firms operate under a less protective EPL, while larger firms are subject to stricter EPL. This is consistent with the political preferences shown in figures 7 and 8. Since increasing EPL harms significantly more the small business sector, a left-wing government keeps weak EPL for those firms and workers. On the other hand, because larger firms can more easily adapt to higher labor costs, the politician sets a stronger level of labor protection for that sector.

The equilibrium threshold, a^x_{pe} is such that it equalizes the weighted marginal workers' benefit and the weighted entrepreneurs' costs at the threshold, as shown by expression (5.6). Figure 10 illustrates proposition 4. It shows the equilibrium labor policy, $\mathcal{P}_{pe}$ as a function of firm's assets a and the politician's orientation, λ. Pro-worker governments set an S-shaped policy as shown by the red dotted line in the figure, while pro-business are not willing to improve EPL and keep low labor protection in all firms, as shown by the blue dashed line.

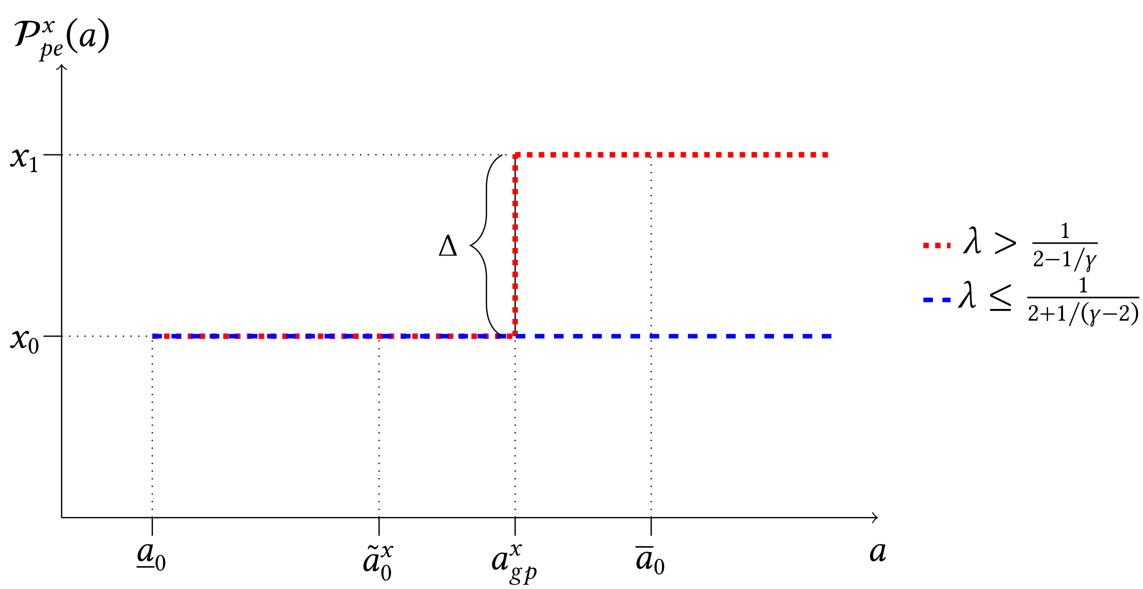

Figure 10: Equilibrium labor policy $\mathcal{P}^x_{pe}$ for $x = \{\varphi, \theta\}$.

An important feature of proposition 4 is that the equilibrium size threshold is defined explicitly for $\lambda \leq \frac{1}{2+1/(\gamma-2)}$ and $\lambda > \frac{1}{2-1/\gamma}$. However, there is no explicit expression for a^x_{pe} when $\lambda \in \left(\frac{1}{2-1/\gamma}, \frac{1}{2+1/(\gamma-2)}\right]$. As explained before, in this case $\bar{U}$ may have more than one peak and these peaks may occur at negative values. The proposition shows that the solution of the politician's problem can be explicitly characterized as long as $\lambda \leq \frac{1}{2+1/(\gamma-2)}$ or $\lambda > \frac{1}{2-1/\gamma}$, i.e., for non-centrist candidates. Section 5.2 shows that when wages are flexible the equilibrium policy can be characterized for any $\lambda \in [0, 1]$.

A final question that should be asked is: what is the effect of λ on the equilibrium size threshold? Intuitively, figure 9 shows that as λ increases, i.e. as the politician becomes more pro-worker, the red solid line receives a larger weight and the maximum of $\bar{U}$ shifts left. Thus, more leftist governments should implement a lower size threshold. That is, more leftist governments establish a more protective EPL. This prediction is consistent with the empirical evidence presented in figures 1 and 2 in section 2. The following lemma formalizes this result.

Lemma 1 *If $\lambda > \frac{1}{2-1/\gamma}$, the equilibrium size threshold, a^x_{pe} under sticky wages is strictly decreasing in λ.*

5.2 Political equilibrium with flexible wages

This section studies the political equilibrium when the equilibrium wage is flexible and responds to changes in the size threshold. This section is divided into three subsections. Subsection 5.2.1 explores the impact of the size threshold on the equilibrium wage. Then, subsection 5.2.2 investigates the political preferences of the different groups of agents when they take into account the effect of shifting the size threshold on the equilibrium wage. Finally, subsection 5.2.3 characterizes the political equilibrium under flexible wages which leads to the main result of the paper.

5.2.1 The size threshold and the equilibrium wage

To start with, the following lemma establishes the effect of a^x on w.

Lemma 2 *The equilibrium wage w is increasing in a^x. In particular, if $a^x = \underline{a}_0$, the change in w is such that $\frac{\partial \bar{w}}{\partial a^x} = 0$.*

The interpretation of lemma 2 is that a less protective labor policy, i.e. a larger a^x, leads to a higher equilibrium wage. The explanation of this result is as follows.

First, suppose that the politician implements a flat labor reform, that is EPL improves from x_0 to x_1 for all firms (i.e. $a^x = \underline{a}_0$). The direct effect of stricter EPL is that the expected wage, $\bar{w}$ is larger. Thus, individual workers supply more labor. Moreover, stronger EPL implies higher operating leverage which crowds out external finance. In consequence, less capital is invested and less labor is demanded. Higher labor supply and lower labor demand imply a lower equilibrium wage. Lemma 2 establishes that the effect of implementing a flat labor policy on the expected labor wage, $\bar{w}$ is exactly counteracted by the reduction in w, i.e. $\bar{w}$ does not change. The intuition is that as long as the net effect on $\bar{w}$ remains positive, workers and firms adjust their labor decisions by pushing down w. This process continues such that in equilibrium the net effect on $\bar{w}$ is zero. Thus, workers' and entrepreneurs' welfare remain unchanged relative to the initial case in which $\mathcal{P}_0 = \{\varphi_0, \theta_0\}$. Thus, when wages are flexible, a flat labor reform is neutral.

Secondly, suppose now that the politician deviates from a flat reform ($a^x = \underline{a}_0$) and marginally increases the size threshold, a^x. Those workers in firms with $a < a^x$ are subject to weaker EPL, and thus, face a lower expected wage, $\bar{w}$. As a result, such workers supply less labor. On the other hand, entrepreneurs operating firms with $a < a^x$ face lower labor costs and then demand more labor. Increased labor demand and reduced labor supply in firms under weaker EPL lead to a higher equilibrium wage relative to the case of a flat reform. As the size threshold increases, the mass of firms facing weaker EPL increases, which leads to a larger w. At the limit, when $a^x \rightarrow a_M$, the equilibrium wage approaches to $w(\mathcal{P}_0)$, i.e., the wage before any regulatory change. In conclusion, increasing the size threshold increases the equilibrium wage. In particular, either passing a flat labor reform ($a^x = \underline{a}_0$) or keeping EPL unchanged ($a^x = a_M$) will maintain economic outcomes unchanged.

Thus, when wages are flexible, the effect of a flat reform is the same as leaving EPL unchanged. That is, $\bar{U}(a^x = \underline{a}_0, \lambda) = \bar{U}(a^x = a_M, \lambda)$ for any λ. The question that must be asked is, can the politician improve welfare ($\bar{U}$) by implementing a size-contingent labor policy?

To answer this question, I start by describing the individual political preferences under a flexible wage. Then, in proposition 5, I characterize the equilibrium labor policy that aggregates these interests.

5.2.2 Political preferences with flexible wages

This subsection characterizes the preferences for EPL of the different groups of workers and entrepreneurs. Figures 11 to 13 depict the change in utilities of the different groups as a function of the size threshold, a^x. The changes are relative to the initial regulation, $\mathcal{P}_0$.

First, figure 11 depicts the change in U^w as a function of the size threshold and for workers matched to a small firm with assets $a < \tilde{a}_0^x$. Section 4 shows that workers in smaller firms experience a decrease in utility when they receive higher protection. However, their utility increases as wages decrease due to increased employment. The greater the decrease in wage, the larger the increase in utility. Thus, when the size threshold is non-binding ($a < a^x$), the change in utility as a function of a^x is positive and decreasing in a^x (since $\frac{\partial w}{\partial a^x} > 0$). On the other hand, because workers in smaller firms suffer from higher protection, there is a discrete fall in utility when the

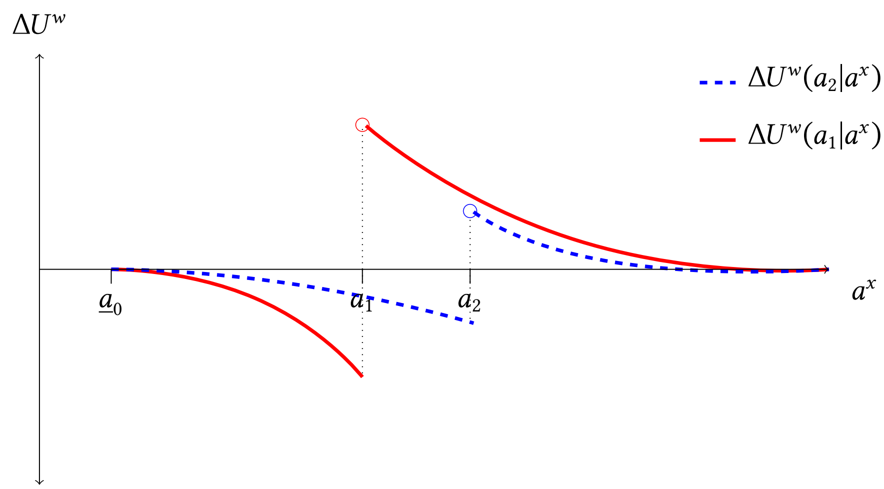

Figure 11: ΔU^w as function of a^x under flexible wages ($a_1 < a_2 < \tilde{a}_0^x$).

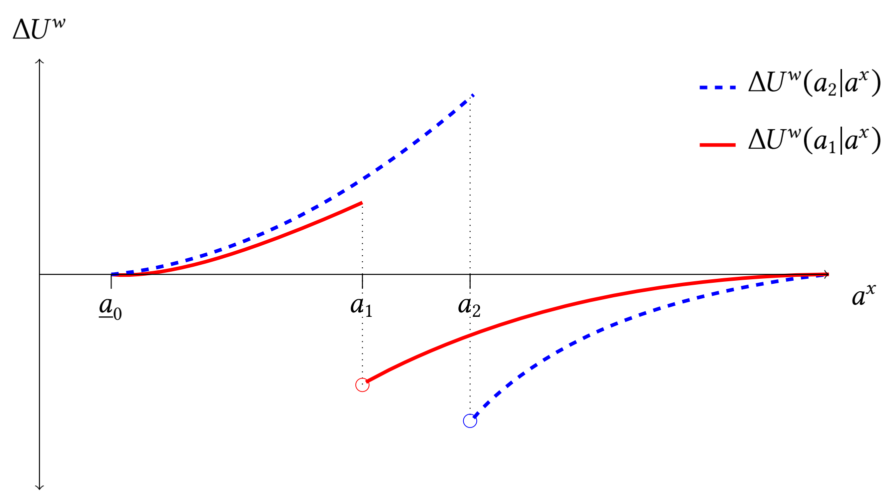

Figure 12: ΔU^w as function of a^x under flexible wages ($a_2 > a_1 > \tilde{a}_0^x$).

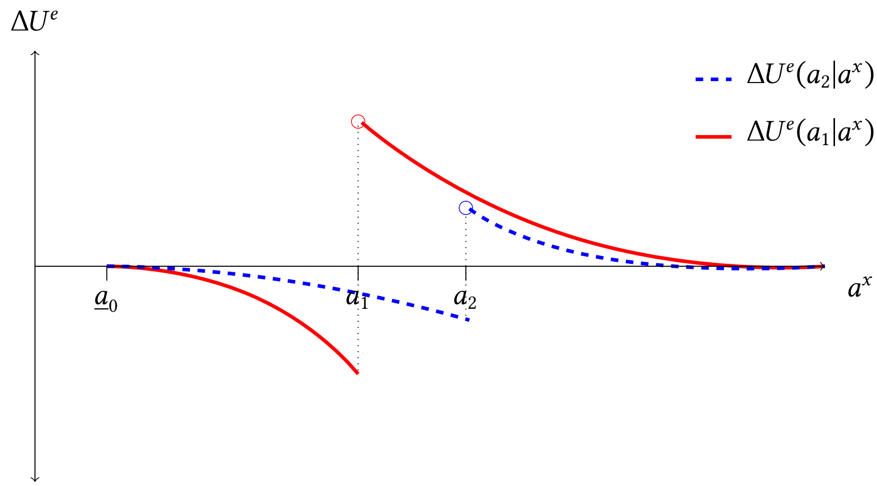

Figure 13: ΔU^e as function of a^x under flexible wages ($a_2 > a_1$).

size threshold becomes binding ($a = a^x$). As a^x declines towards $\underline{a}_0$, the change in utility returns to zero.

Figure 11 also compares the utility gains of workers matched to firms of different sizes, a_1 and a_2 ($a_1 < a_2 < \tilde{a}_0^x$). The red solid line shows that workers in less capitalized firms (a_1) benefit more from a non-binding size threshold ($a_1 < a^x$). Conversely, the blue dashed line shows that workers in more capitalized firms (a_2) suffer less from being subject to stricter EPL ($a_2 \geq a^x$).

Secondly, figure 12 shows the change in utility of workers matched to larger firms ($a > \tilde{a}_0^x$). The effects are reversed relative to figure 11. As discussed in section 4, these workers benefit from a higher wage and better protection. In this case, workers in larger firms (a_2) benefit more from increased protection (blue dashed line), while those in less capitalized firms (a_1) are less affected by not receiving that higher protection (red solid line).

Thirdly, figure 13 shows the change in entrepreneurs utilities as a function of a^x. Entrepreneurs benefit from stricter EPL as long as they are not subject to its regulations ($a < a^x$). The explanation is that a more protective EPL, that is, a lower size threshold, decreases the equilibrium wage and reduces operational costs. However, when entrepreneurs are subject to stricter EPL ($a > a^x$), they utility decreases as they must pay a higher expected wage, $\bar{w}$. As illustrated in the figure, entrepreneurs operating less capitalized firms (a_1) benefit more from being excluded from higher protection (red solid line), while those running larger firms (a_2) suffer less from facing more stringent EPL (blue dashed line).

To sum up, there are conflicting interests regarding EPL. Workers in smaller firms ($a < \tilde{a}_0^x$) would prefer stricter EPL for everyone except themselves. Meanwhile, workers in larger firms ($a > \tilde{a}_0^x$) would prefer high protection for themselves but not for others. All firms would like strong EPL for their competition but to operate under weak EPL themselves. The questions that remain are, what is the best EPL design that balances these political interests and how does it depend on the political orientation of the government?

Intuitively, based on figures 11 and 12, a left-wing government may want to impose an S-shaped EPL because it can benefit both workers in large ($a > \tilde{a}_0$) and small firms ($a < \tilde{a}_0$). However, in choosing the labor policy, the government must balance two opposing forces: decreasing the size threshold benefits workers in smaller firms, but hurts those in larger firms due to reduced wages. On the other hand, figure 13 suggests that a right-wing government can benefit owners of smaller firms by imposing stricter EPL on larger firms. Next section characterizes the equilibrium policy when wages are flexible.

5.2.3 Equilibrium policy with flexible wages

Now I proceed to the study of the equilibrium labor policy under flexible wages. To simplify the exposition define,

$$\bar{U}^e(a^x) \equiv \int_{\underline{a}}^{a^x} U^e(a|x_0)\partial G + \int_{a^x}^{a_M} U^e(a|x_1)\partial G, \tag{5.7}$$

$$\bar{U}^w(a^x) \equiv \int_{\underline{a}}^{a^x} U^w(a|x_0)\partial G + \int_{a^x}^{a_M} U^w(a|x_1)\partial G, \tag{5.8}$$

where expression (5.7) is the aggregate entrepreneurs' welfare ($\lambda = 0$) and (5.8) corresponds to the aggregate workers' welfare ($\lambda = 1$). Thus, the asset-based welfare is written as

$$\bar{U}(a^x, \lambda) = \lambda \cdot \bar{U}^w(a^x) + (1 - \lambda) \cdot \bar{U}^e(a^x). \tag{5.9}$$

The following proposition characterizes the political equilibrium.

Proposition 5

1. *$\bar{U}(a^x, \lambda)$ achieves a global maximum in $[\underline{a}_0, a_M]$ at some size threshold $a^x_{pe} \in (\underline{a}_0, a_M)$ characterized by*

$$a^x_{pe} = \sup_{a^x} \bar{U}(a^x, \lambda). \tag{5.10}$$

Suppose that $\mathrm{g}(\cdot)$ satisfies $\mathrm{g}' < 0$, then,

2. *$\bar{U}^e(a^x, \lambda)$ and $\bar{U}^w(a^x, \lambda)$ are strictly concave in a^x.*

3. *The equilibrium size threshold a^x_{pe} under flexible wages is the unique solution to,*

$$\lambda \frac{\partial \bar{U}^w(a^x_{pe}, \lambda)}{\partial a^x} = -(1-\lambda)\frac{\partial \bar{U}^e(a^x_{pe}, \lambda)}{\partial a^x}, \quad x \in \{\varphi, \theta\}. \tag{5.11}$$

4. *The equilibrium size threshold a^x_{pe} is decreasing in λ.*

The proposition shows that, without imposing any additional assumption on the wealth distribution function $g(\cdot)$, the political equilibrium is such that the size threshold satisfies: $a^x \in (\underline{a}_0, a_M)$. That is, the equilibrium EPL is S-shaped regardless of political orientation of the government, λ. As opposed to the case with sticky wages, the equilibrium policy can be characterized by (5.10) for any $\lambda \in [0, 1]$.

Under the additional assumption that $g' < 0$, both $\bar{U}^e$ and $\bar{U}^w$ are strictly concave in the size threshold a^x. Thus, $\bar{U} = \lambda\bar{U}^w + (1-\lambda)\bar{U}^e$ is concave for any $\lambda \in [0, 1]$. The equilibrium EPL is uniquely given by (5.11) for any λ. Figure 14 illustrates these features. The red solid line corresponds to $\bar{U}^w(a^x, \lambda = 1)$, where a^x_{LW} is the equilibrium policy when $\lambda = 1$ (left-wing). The blue dashed line shows $\bar{U}^e(a^x, \lambda = 0)$ which reaches its maximum at some a^x_{RW} (right-wing). The dotted line corresponds to $\bar{U}(a^x, \lambda)$ for $\lambda \in (0, 1)$ which attains its maximum at some $a^x_C \in (a^x_{LW}, a^x_{RW})$.

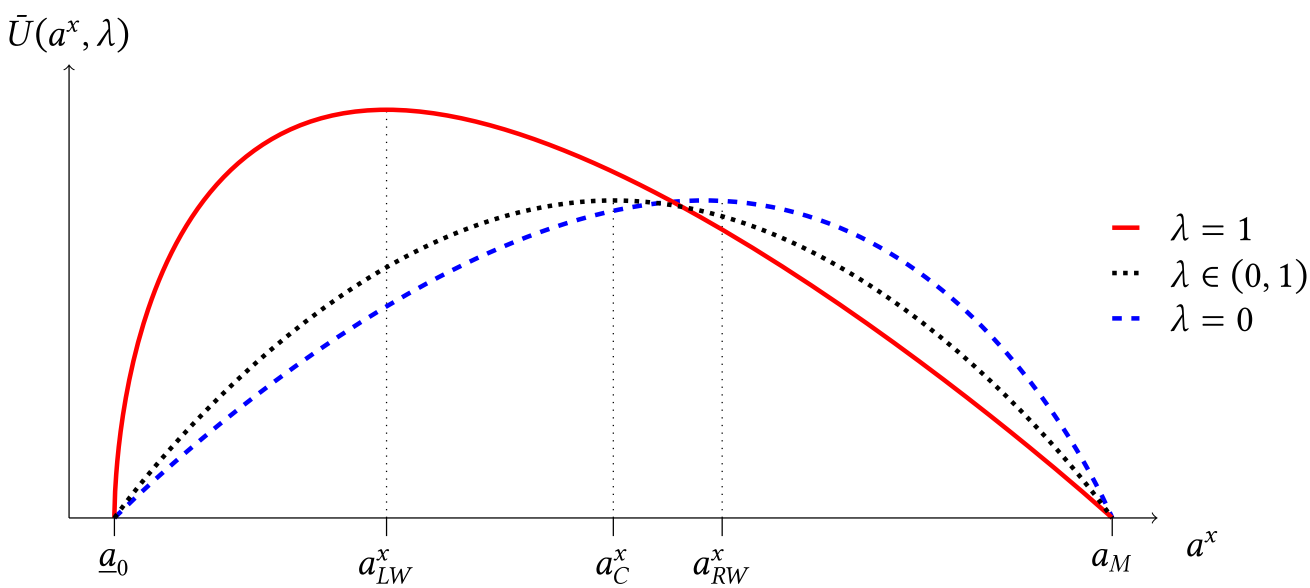

Figure 14: Asset-based welfare ($\bar{U}$) as a function of λ and a^x. Flexible wage and $g' < 0$.

Proposition 5 states the main result of the paper. The equilibrium EPL under flexible wages is S-shaped regardless of the political orientation of the government. Thus, even when the government cares only about entrepreneurs, it imposes stricter EPL on larger firms. Conversely, even when the government cares only about workers, it keeps workers in smaller firms under less protection. Moreover, the size threshold is decreasing in λ, thus more leftist governments establish a more protective EPL. These results are consistent with the stylized facts presented in figures 1 and 2 in section 2.

The intuition for these results is as follows. First, right-wing governments understand that stricter EPL in larger firms leads to a lower equilibrium wage due to increased competition in the labor market. The small-scale sector significantly benefits from lower labor costs due to increased access to credit and investment. Large firms have to pay higher labor costs, but can more easily adjust their operations due to unconstrained access to credit. Thus, from a right-wing government's perspective, an S-shaped EPL is a way to cross-subsidize smaller firms at a relatively low cost for larger firms.

Secondly, left-wing governments understand that smaller firms cannot accommodate stricter EPL, which would negatively affect their workers. Thus, even when a left-wing government would like to give protection to all workers, it keeps those in smaller firms under weak protection to protect them from the negative impact that EPL would have on their firms' operations.

Finally, when λ increases, i.e. when the government is more leftist, the red solid line in figure 14 receives a larger weight relative to the dashed blue line. Thus, the maximum of the dotted line a_C^x moves to the left. In consequence, more leftist governments establish a lower size-threshold.

To sum up, the political motivation of a right-wing government to establish an S shape EPL can be stated as follows:

> "*regulate large businesses to foster small businesses growth*",

while the ideal of a left-wing government is:

> "*do not regulate the small businesses to protect their workers*".

5.3 Discussion: sticky versus flexible wages

In this section, I briefly discuss the differences between the equilibrium policies under sticky and flexible wages. First, section 5.1 shows that, when wages are sticky, only more leftist governments are willing to implement an S-shaped EPL. From the point of view of more right-wing governments, increasing EPL is too costly for firms. Thus, they keep low EPL across the board. On the other hand, section 5.2 shows that when wages are flexible, firms that are not subject to stricter EPL benefit from reduced wages. In that case, right-wing governments are willing to impose stricter EPL to larger firms as a way to cross-subsidize the small business sector. Left-wing governments keep smaller firms under weak EPL to protect their workers, so they also implement an S-shaped EPL.

Based on these results, one should expect that S-shaped EPLs are more likely to emerge in countries where wages are more flexible and under more leftist governments. In contrast, in countries where wages are more rigid (e.g. high minimum wages) the ability of wages to offset the effects of EPL is more limited. Thus, governments are less likely to impose an S-shaped EPL in such countries. Related to these results, Garicano et al. (2016) show that aggregate welfare losses from S-shaped regulations are increasing in the degree of wage rigidity.

5.4 Ex-post competitive equilibrium

This section characterizes the ex-post competitive equilibrium that arises as a result of implementing the labor policy described in section 5.2, denoted by $\mathcal{P}$. First, as a result of more protective EPLs, there is stronger competition in the labor market. Thus, the equilibrium wage is lower than under $\mathcal{P}_0$, $w(\mathcal{P}) < w(\mathcal{P}_0)$. From the point of view of individual workers, those working for firms with $a \geq a^x$ receive an expected wage $\bar{w}^1 \equiv \bar{w}(\mathcal{P}|a \geq a^x)$ larger than the one under initial regulations $\bar{w}(\mathcal{P}_0)$. In contrast, those in firms with $a < a^x$ are paid a lower expected wage, $\bar{w}^0 \equiv \bar{w}(\mathcal{P}|a < a^x) < \bar{w}(\mathcal{P}_0)$.

Suppose a relatively protective labor policy, such that $a^x < \underline{a}$. From the point of view of firms, those such that $a \in [\underline{a}, a^x)$ face lower labor costs after a regulatory change and thus, have easier access to credit and operate at a more efficient scale. On the other hand, those credit constrained firms ($a \in [a^x, \overline{a})$) that are subject to stricter EPLs, suffer from higher operating costs, lose access to credit and thus have to shrink. More capitalized firms ($a > \overline{a}$) remain unconstrained and continue operating optimally even when they pay higher expected wages. Figure 15 illustrates the ex-post competitive equilibrium.

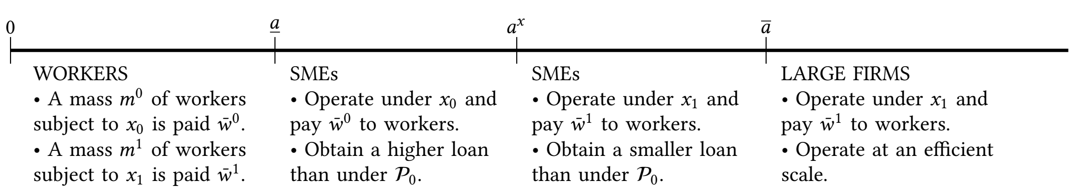

Figure 15: Ex-post competitive equilibrium.

The figure shows the ex-post equilibrium that will arise under an EPL design more likely to be implemented by a left-wing government ($a^x < \overline{a}$). In contrast, a more right-wing government may want to implement a threshold such that $a^x \geq \overline{a}$. In that case, all firms with $a < \overline{a}$ benefit from the a regulatory change, while unconstrained entrepreneurs with $a \geq a^x$ bear the costs.[27]

[27]Note that the ex-post welfare of each group of agents will depend on the political orientation of the government, λ. In section D.3 in the Appendix, I study the political affiliations of the different groups of agents if they can anticipate the EPL to be implemented by a leftist and a right-wing government.

6 Extensions

This section presents two extensions of the baseline model from section 3. In section 6.1, I examine the equilibrium EPL when firm size is determined by labor, as in the data. In section 6.2, I investigate the EPL that results from independent negotiations between workers and entrepreneurs. In section D.5 of the Appendix, I briefly explore the distortions generated when agents can self-report their assets.

6.1 Labor-based policy

This section studies a more realistic environment where politicians can choose to apply regulations contingent on labor. In response to a labor-based policy, a group of firms strategically hire their labor creating distortions on welfare. The main takeway is that politicians account for these distortions and still decide to implement an S-shaped EPL, as observed in the data. However, as a result of these distortions, the ability of an S-shaped policy to generate "cross subsidies" through wages is deteriorated. In consequence, the labor-based welfare is lower than the asset-based welfare obtained in section 5, when there was no strategic behavior.

6.1.1 The problem of politicians

The labor regulation $\mathcal{P} = (\mathcal{P}^{\varphi}, \mathcal{P}^{\theta})$ is a function that maps labor to a specific strengh of EPL. Formally, $\mathcal{P}^{\varphi}(l) : [l^{\varphi}_{min}, l^{\varphi}_{max}] \to \{\varphi_0, \varphi_1\}$ and $\mathcal{P}^{\theta}(l) : [l^{\theta}_{min}, l^{\theta}_{max}] \to \{\theta_0, \theta_1\}$. As before, I denote $x \in \{\varphi, \theta\}$. Recall that the optimal labor function is increasing in a and decreasing in x. Thus, the domain of $\mathcal{P}^x$ is defined by $l^x_{min} = l(\underline{a}_0|x_1)$ and $l^x_{max} = l(\overline{a}_0|x_0)$.[28]

The problem of the politician is,

$$\begin{aligned} &\max_{\{\mathcal{P}(l)\}_{l_{min}}^{l_{max}}} \{\tilde{U}(\mathcal{P}) \equiv \lambda \cdot \mathbb{E}_G[U^w|\mathcal{P}] + (1-\lambda) \cdot \mathbb{E}_G[U^e|\mathcal{P}]\} \\ &s.t. \quad \mathbb{E}_G[l_s|\mathcal{P}] = \mathbb{E}_G[l|\mathcal{P}]. \end{aligned} \quad , \tag{6.1}$$

As in the case of an asset-based policy, it can be shown that the solution to this problem satisfies monotonicity in both components. Proposition 8 in section D in the Appendix shows this result. Thus, there is a labor threshold $l^x \in [l^x_{min}, l^x_{max}]$ above which EPL becomes stricter.

[28]Note $l(\overline{a}_0|x_0)$ is the maximum amount of labor any firm would ever hire. Conversely, $l(\underline{a}_0|x_1)$ is the minimum amount of labor a firm would hire.

Formally,

$$\mathcal{P}^x(l) = \begin{cases} x_0 & \text{if } l < l^x, \\ x_1 & \text{if } l \geq l^x. \end{cases} \tag{6.2}$$

To simplify exposition, in what follows I study a regulatory change in a single dimension. Thus, politicians consider increasing either individual or collective dismissal regulation, but not both at the same time.

6.1.2 Strategic behavior

In response to a labor-based policy as in equation (6.2), firms strategically choose how much labor to hire. They can legally avoid being hit by EPL by hiring an amount of labor just below l^x. More specifically, there is an endogenous range of firms $[a_1^x, a_2^x]$ that hire slightly less labor than l^x to operate under weak EPL. Formally, these two thresholds are defined as follows,

$$U^e(a_1^x, d(a_1^x), l^x | x_0) = U^e(a_1^x, d(a_1^x), l(a_1^x) | x_0), \tag{6.3}$$

$$U^e(a_2^x, d(a_2^x), l^x | x_0) = U^e(a_2^x, d(a_2^x), l(a_2^x) | x_1), \tag{6.4}$$

where the asset thresholds a_1^x and a_2^x are implicit functions of l^x. As evidence of such strategic behavior, Gourio and Roys (2014) and Garicano et al. (2016) show that the firm size distribution is distorted in France, where firms with 50 employees or more face substantially stricter labor regulations. In particular, few firms have exactly 50 employees, while a large number of firms have 49 employees.

Figure 16 illustrates the units of labor hired as a function of assets given a labor regulation $\mathcal{P}^x$. There are three groups of firms. First, firms with $a \in [\underline{a}_0, a_1^x)$ are subject to weak EPL (x_0) and hire labor optimally. Secondly, firms with $a \in [a_1^x, a_2^x]$ hire slightly less than l^x units of labor in order to operate under weak EPL. Thus, they hire less labor than what is optimal according to their operation scale.[29] Finally, firms with $a > a_2^x$ operate under stricter EPL (x_1) and hire an optimal amount of labor given their investment level.

[29] Recall that given capital, $k(a|x) = a + d(a|x)$, the optimal amount of labor given the strength of EPL x ($l(a|x)$) is given by $p f_l(k(a|x), l(a|x)) = \bar{w}(x)$. Firms that belong to $(a_1^x, a_2^x]$ hire a less labor than what is optimal given their capital, thus $p f_l(k, l^x) > \bar{w}(x)$.

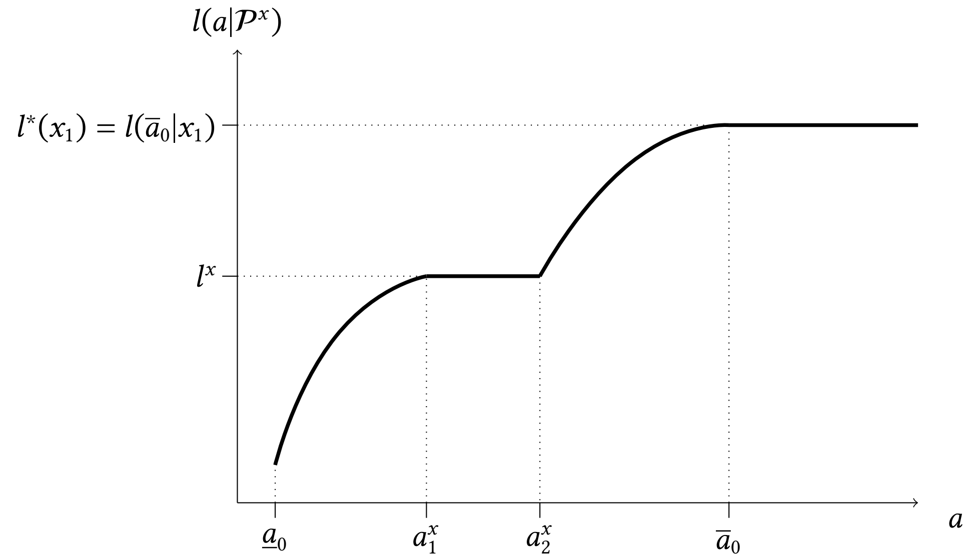

Figure 16: Labor decisions as function of assets.

6.1.3 Political equilibrium under a labor-based policy

Equation (6.2) and conditions (6.3) and (6.4) allow me to write the politicians' problem more explicitly. Define the total entrepreneurs' and workers' welfare as follows,

$$\tilde{U}^e(l^x) = \int_{\underline{a}_0}^{a_1^x} U^e(a, l(a)|x_0)\partial G(a) + \int_{\boldsymbol{a_1^x}}^{\boldsymbol{a_2^x}} \boldsymbol{U^e(a, l^x|x_0)\partial G(a)} + \int_{a_2^x}^{a_M} U^e(a, l(a)|x_1)\partial G(a), \tag{6.5}$$

$$\tilde{U}^w(l^x) = \int_{\underline{a}_0}^{a_1^x} U^w(a, l(a)|x_0)\partial G(a) + \int_{\boldsymbol{a_1^x}}^{\boldsymbol{a_2^x}} \boldsymbol{U^w(a, l^x|x_0)\partial G(a)} + \int_{a_2^x}^{a_M} U^w(a, l(a)|x_1)\partial G(a), \tag{6.6}$$

where the bold terms capture the direct welfare distortions generated by strategic behaviour of firms.[30]

Then, the problem of the politician is,

$$\max_{l^x \in [0, l^x_{max}]} \{\tilde{U}(l^x) = \lambda \tilde{U}^w(l^x) + (1-\lambda)\tilde{U}^e(l^x)\}$$

$$s.t \qquad m^0 \cdot l_s(x_0) = \int_{\underline{a}_0}^{a_1^x} l(a|x_0)\partial G(a) + l^x \cdot [G(a_2^x) - G(a_1^x)], \tag{6.7}$$

$$m^1 \cdot l_s(x_1) = \int_{a_2^x}^{a_M} l(a|x_1)\partial G, \tag{6.8}$$

$$m^0 + m^1 = G(\underline{a}_0), \tag{6.9}$$

[30]Note that these distortions also create a general equilibrium effect through wages. Thus, these distortions also have an impact on the utilities of the rest of the agents that don't act strategically.

where equations (6.7) to (6.9) are the labor market conditions. Note that politicians now choose regulations while taking into account the distortions that strategic behavior creates on welfare and on the labor market. In section D.4 in the Appendix, I show that the politician's problem can be mapped into a problem in which she chooses an asset threshold a_1^x to maximize the labor-based welfare. Once the problem is rewritten in terms of an asset threshold, the same insights described in section 5 apply. Proposition 9 in the Appendix shows that the equilibrium policy is still S-shaped regardless of the political orientation of the government. That is, there is a size threshold l^x such that firms hiring more than l^x units of labor face stricter EPL. This result is consistent with the empirical evidence presented in section 2.

Overall, when EPL is defined in terms of labor, politicians have to take into account the distortions generated by strategic behavior. The fact that a group of firms hire slightly less labor than l^x implies that the equilibrium wage decreases by less when EPL becomes more protective (i.e. l^x decreases). Thus, the ability of the government to generate "cross subsidies" by proposing an S-shaped regulation is deteriorated. In consequence, the labor-based welfare is lower than the asset-based welfare obtained in section 5, when there was no strategic behavior. The final question is whether there is an alternative mechanism that survives strategic behavior and that implements the maximum asset-based welfare. Next section proposes such alternative mechanism.

6.2 Bargaining

This section presents an alternative mechanism through which governments can achieve the maximum asset-based welfare: independent bargaining between workers and entrepreneurs. Each group of workers in each firm is organized as a union. That is, as a society with the purpose of promoting working conditions in line with their common interests. Unions bargain with the owners of their firms (entrepreneurs) to define EPL before production takes place and to maximize their workers' welfare, U^w. Politicians can control the resulting outcome of negotiations by regulating unions' bargaining power. The policy instrument–unions' bargaining power–is a single dimensional parameter which is uniform across firms. Thus, it survives strategic behavior of firms. That is, firms cannot adjust their size in order to face more favorable regulations. The main result of this section is that, under some conditions, governments can implement the maximum asset-based welfare by properly allocating the bargaining power between unions and firms.

6.2.1 Timeline

Figure 17 illustrates the timeline. At $t = 0$, workers are randomly matched to a firm given initial regulations $\mathcal{P}_0$. The different groups of workers form unions to bargain on EPL with their firms.

Negotiation terms are as follows. At $t = 1$, potential entrepreneurs and unions sign an employment contract which defines the strength of EPL to be exercised at $t = 2$. The contract specifies whether the firm is going to operate under weak EPL (x_0) or strong EPL (x_1). Entrepreneurs cannot precommit to a given level of employment since debt and labor are decided at period $t = 2$, i.e. after $\mathcal{P}$ has been set. Conversely, at $t = 1$, unions in bargaining with entrepreneurs set their demands taking into account the effect on debt, and thus, on the amount of labor that will be hired at $t = 2$. However, as negotiations take place independently between unions and entrepreneurs of different firms, they cannot anticipate the general equilibrium effects of the economy-wide changes in EPL. At $t = 2$, the economy operates under the new EPL $\mathcal{P}$ that results from independent negotiations. Production takes place and loans are repaid.

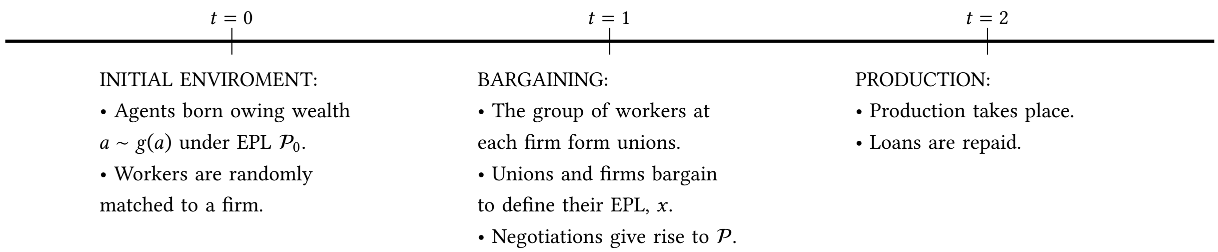

Figure 17: Timeline.

6.2.2 Bargaining mechanism

Unions and entrepreneurs bargain over the firm's specific EPL by following the random proposer model by Binmore (1987). Unions and entrepreneurs make take-it-or-leave-it proposals with frequencies μ and $1 - \mu$, respectively. That is, a firm's EPL is set at the union's optimal level with frequency μ and at the entrepreneur's preferred level with frequency $1 - \mu$. Thus, $\mu \in [0, 1]$ can be interpreted as the unions' bargaining power, which is now the unique policy instrument of the government.

Importantly, μ is not size-contingent. That is, the government's policy intervention operates in a single dimension and it is uniform across firms. This means that firms cannot strategically adjust their size in order to face less stringent regulations. Since the government's policy has a single degree of freedom, it is a non-trivial task to find a level of μ that can replicate the maximum asset-based welfare of section 5.

6.2.3 Equilibrium policy

Negotiations lead to the expected labor regulation policy, $\mathcal{P}_{rp} : [\underline{a}_0, a_M] \to \mathcal{O}$ to be implemented at $t = 2$, where $\mathcal{O}$ is the convex set given by:

$$\mathcal{O} = \{(\varphi, \theta) : \left(\zeta^{\varphi}\varphi_0 + (1 - \zeta^{\varphi})\varphi_1, \zeta^{\theta}\theta_0 + (1 - \zeta^{\theta})\theta_1\right); \; \zeta^{\varphi}, \zeta^{\theta} \in [0, 1]\}.$$

Lemma 3 *The expected labor regulation policy, $\mathcal{P}_{rp} : [\underline{a}_0, a_M] \to \mathcal{O}$ that arises from the random proposer model satisfies,*

$$\mathcal{P}^{\varphi}_{rp}(a) = \begin{cases} \varphi_0 & \text{if } a \in [\underline{a}_0, \tilde{a}^{\varphi}_0), \\ \varphi_0 + \mu\Delta & \text{if } a \geq \tilde{a}^{\varphi}_0, \end{cases} \tag{6.10}$$

and

$$\mathcal{P}^{\theta}_{rp}(a) = \begin{cases} \theta_0 & \text{if } a \in [\underline{a}_0, \tilde{a}^{\theta}_0), \\ \theta_0 + \mu\Delta & \text{if } a \geq \tilde{a}^{\theta}_0. \end{cases} \tag{6.11}$$

Figure 18 illustrates lemma 3. As opposed to section 5, the government has no control over the size threshold at which EPL becomes stricter, $\tilde{a}^x_0$. In this case, the government can alter the equilibrium policy by changing the bargaining power of unions, μ. Thus, now the government has control over the size of the discontinuity at the size threshold. In what follows I show the conditions under which the expected regulation policy that arises from the random proposer model can replicate maximum asset-based welfare.

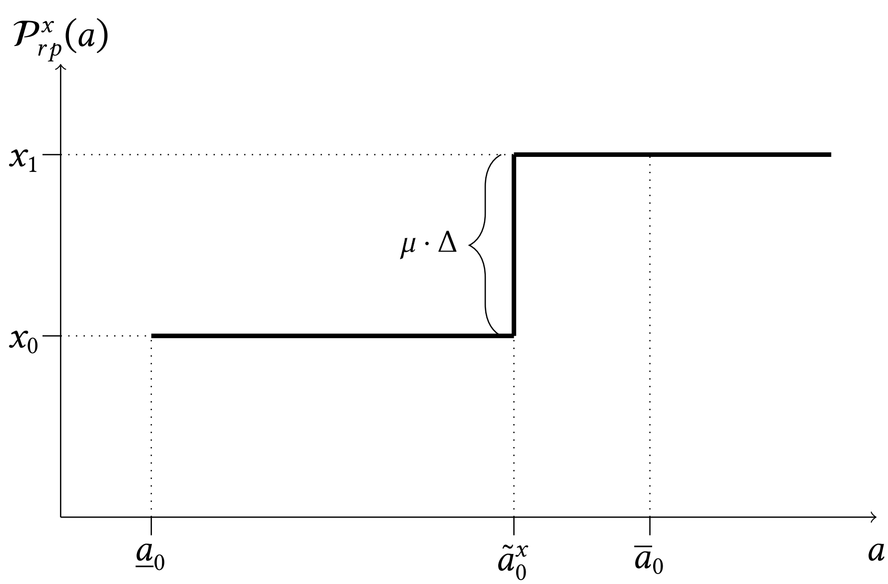

Figure 18: Expected labor regulation policy, $\mathcal{P}^x_{rp}$ for $x = \{\varphi, \theta\}$.

6.2.4 Bargaining under sticky wages

I analyze the case in which wages are sticky which is simpler. The question to be studied in this section is as follows. Can the government choose unions' bargaining power such that the resulting expected labor policy replicates the maximum asset-based welfare?

This question translates into finding a μ such that $\mathcal{P}_{rp}$ gives same welfare given by the threshold that maximizes the asset-based welfare (equation (5.6) in section 5.1.2). The following proposition characterizes the bargaining power of unions, $\mu(\lambda)$ that replicates the maximum asset-based welfare given the government's political orientation (λ).

Proposition 6 *The union's bargaining power function, $\mu(\lambda)$ that implements the maximum asset-based welfare is as follows,*

$$\mu(\lambda) = \begin{cases} 0 & \text{if } \lambda \leq \frac{1}{2+1/(\gamma-2)}, \\ \chi(\lambda) & \text{if } \lambda \in (\tilde{\lambda}, 1], \end{cases} \tag{6.12}$$

where $\chi(\lambda) \in (0, 1]$ is some increasing function in λ such that $\chi(1) = 1$ and $\tilde{\lambda} > \frac{1}{2-1/\gamma}$.

The proposition shows that there is a union's bargaining power that can implement the maximum asset-based welfare of the preferred policy for $\lambda \in \left[0, \frac{1}{2+1/(\gamma-2)}\right] \bigcup (\tilde{\lambda}, 1]$. As expected, more leftist governments provide higher bargaining power to unions. In contrast, right-wing governments are able to exactly enforce their preferred policy by not allowing unions to exist, $\mu = 0$. Left-wing regulators can implement the exact equilibrium policy only when $\lambda = 1$ and by giving all the bargaining power to unions, $\mu = 1$. Otherwise, when $\lambda \in (\tilde{\lambda}, 1)$, the maximum asset-based welfare is achievable under a labor policy that is different to the one described in section 5.1. In what follows I explain the intuition for this last result.

In this case the government does not have control over the threshold above which EPL becomes stricter, which is now fixed and given by $\tilde{a}_0^x$. However, section 5.1.2 shows that, when $\lambda \in (\tilde{\lambda}, 1)$, the preferred policy is such that the size threshold satisfies: $a^x > \tilde{a}_0^x$. Thus, when $\lambda \in (\tilde{\lambda}, 1)$, the labor policy arising from independent negotiations has a lower size threshold than the preferred policy. That is, it provides protection to a larger set of workers. The government can solve this issue by limiting the bargaining power of unions (μ), that is by controlling the intensive margin of EPL. In figure 18 this means reducing the size of the 'jump' ($\mu\Delta$) at the threshold. As a result, the policy that implements the maximum asset-based welfare provides protection to a larger set of workers, but the intensity of this protection is lower.

The main takeaway of this section is that the government can eliminate the distortions created by strategic behavior by properly allocating the bargaining power between workers and entrepreneurs. The explanation for this result comes from the fact that in equilibrium there are no unions in smaller firms ($a < \tilde{a}_0^x$). That is, even when workers from this sector are allowed to form unions and bargain on labor conditions, they accept to remain under weak protection regardless of their bargaining power. Thus, is like unions never come to exist in smaller firms. In consequence, the government chooses μ to control the outcome of negotiations in larger firms ($a > \tilde{a}_0^x$) and in this way, attain the desired level of welfare.

7 Conclusions

Employment protection legislation (EPL) is a set of rules that govern the termination of job contracts. These regulations take different forms, such as severance payments, reinstatement possibilities, and notification procedures. In many countries, EPL is not equally enforced across firms. Firms with the number of employees higher than a certain threshold face stricter dismissal regulations (S-shaped EPL). The quantitative macro literature shows that the welfare costs of EPL can amount to 3.5% of GDP. But if EPL is so costly, why it exists and why is S-shaped in many countries?

To address these questions, this article develops a political and economic theory that rationalizes the emergence of S-shaped EPL. Citizens are heterogeneous in wealth and choose to become workers or entrepreneurs. Occupational choice determines their voting preferences for EPL. The equilibrium policy is defined by a probabilistic voting model à la Persson and Tabellini (2000). Initially, workers in all firms are poorly protected against dismissal, so EPL is said to be weak. Two political candidates make a binary decision for each firm: whether to keep the initially weak EPL or to apply stricter EPL. Thus, the equilibrium EPL can be potentially size-contingent.

I start with a baseline model in which politicians define policies in terms of observable assets (asset-based policy), and thus, EPL is enforceable. Then, I study a more realistic environment where politicians implement a labor-based policy. In both cases, the equilibrium policy maximizes a politically-weighted welfare. The weights depend on a parameter measuring the government's political orientation.

The main result is that the equilibrium policy is S-shaped, regardless of the government's primary concern for either workers or entrepreneurs. This result applies to both an asset-based or labor-based policy. However, under a labor-based policy, firms strategically adjust their size to legally avoid being hit by regulations. As a result of these distortions, the labor-based welfare is lower than the asset-based welfare. Under some conditions, I show that politicians can implement the maximum asset-based welfare by allowing the groups of workers (unions) and entrepreneurs to bargain on regulations. Therefore, politicians can eliminate the distortions caused by strategic behavior by properly allocating the bargaining power between unions and entrepreneurs.

The rationale for the emergence of an S-shaped EPL comes from its effects on the labor market. Higher worker protection in larger firms increases labor market competition, which results in a lower equilibrium wage. Smaller firms substantially benefit from reduced wages, while larger firms can more easily absorb stricter EPL. Thus, pro-business governments view an S-shaped EPL as a way to cross-subsidize small firms at a relatively low cost for larger firms. On the other hand, pro-worker governments anticipate that smaller firms would struggle to accommodate stricter EPL, which in turn would negatively affect their workers. Thus, they also impose lighter EPL on smaller firms.

This paper opens the door for a deeper understanding of the emergence of labor regulation across countries. First, as far as I know, this is the first paper to develop a political theory where an S-shaped EPL can arise as an equilibrium outcome of aggregating endogenous political preferences.

Secondly, the model provides new testable predictions regarding the welfare effects of EPL across groups of workers and firms. EPL, which supposedly protects workers, reduces the welfare of the group of workers in smaller firms while only benefiting those in larger firms. Moreover, larger firms suffer less from EPL than smaller firms. In a companion paper (Huerta, 2022), I provide empirical support for these results by using firm-level panel data and by exploiting the state-level adoption of Wrongful Discharge Laws (WDLs) in the US. Thirdly, the model shows that more protective labor regulations are more likely to arise in countries with more flexible wages, which is a new result that can be tested in the data.

Finally, other types of size-contingent regulations are widespread worldwide, such as special tax treatments, credit subsidies, and restrictions on the expansion of businesses. As shown in section D.6 in the Appendix, the model can be adapted to accommodate these different policies. The study of the political economy of these regulations is left for future work.

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A Appendix: Basics

A.1 Optimal debt contract

In what follows I characterize the conditions that define the optimal debt contract under the initial policy, $\mathcal{P}_0 = (\varphi_0, \theta_0)$. These conditions can be generalized to any policy, $\mathcal{P}$.

Define the auxiliary function,

$$\Psi(a, d, l|\varphi_0, \theta_0) \equiv U^e(a, d, l|\varphi_0, \theta_0) - \phi k, \tag{A.1}$$

which measures the severity of agency problems for a triplet (a, d, l).[31] Analogously as in FH, it can be shown that there exists a minimum wealth required to obtain a loan, $\underline{a}_0 = \underline{a}(\varphi_0, \theta_0)$ which is given by[32]

$$\Psi(\underline{a}_0, \underline{d}_0, \underline{l}_0|\varphi_0, \theta_0) = 0 \Leftrightarrow U^e(\underline{a}_0, \underline{d}_0, \underline{l}_0|\varphi_0, \theta_0) = \phi \underline{k}_0 \tag{A.2}$$

$$\Psi_d(\underline{a}_0, \underline{d}_0, \underline{l}_0|\varphi_0, \theta_0) = 0 \Leftrightarrow p f_k(\underline{k}_0, (1-s)\underline{l}_0) = 1 + r^* + \phi, \tag{A.3}$$

$$\frac{\partial U^e(\underline{a}_0, \underline{d}_0, \underline{l}_0|\varphi_0, \theta_0)}{\partial l} = 0 \Leftrightarrow p f_l(\underline{k}_0, (1-s)\underline{l}_0) = \bar{w}(\varphi_0, \theta_0), \tag{A.4}$$

where $\underline{k}_0 \equiv \underline{a}_0 + \underline{d}_0$, $\underline{d}_0 > 0$ is the amount of debt that the first agent with access to credit can get and $\underline{l}_0$ are the units of labor he hires. Intuitively, the first condition asks that the minimum wealth to get a loan $\underline{a}_0$ leaves the agent just indifferent between absconding with the loan or honoring the contract. The second expression imposes that an agent with $\underline{a}_0$ is obtaining his minimum debt, $\underline{d}_0$. The final condition imposes that labor hired $\underline{l}_0$ is optimal at the capital level $\underline{a}_0 + \underline{d}_0$.

Thus, there is credit rationing: a rationed borrower ($a < \underline{a}_0$) may be willing to pay a higher interest rate in order to obtain a loan, but investors will not accept such offer since they cannot trust the borrower. From condition (A.3), the marginal return to investment of the first agent with access to credit is $1 + r^* + \phi$, which corresponds to the highest possible return to investment. As a increases, the return to capital falls until eventually it attains the level obtained by an efficient firm $1 + r^*$. Since U^e is increasing and continuous in the relevant range, there exists a critical wealth level $\overline{a}_0 > \underline{a}_0$, such that an entrepreneur owing $\overline{a}_0$ is the first agent that can obtain a loan to invest efficiently,

$$\Psi(\overline{a}_0, k_0^* - \overline{a}_0, l^*) = 0. \tag{A.5}$$

Thus, in equilibrium these two thresholds define an endogenous range of entrepreneurs, $[\underline{a}_0, \overline{a}_0)$

[31] If $\Psi > 0$ the incentives to commit default decrease as Ψ increases. In contrast, if $\Psi < 0$ the entrepreneur has incentives to behave maliciously. A more negative Ψ means that the entrepreneur has less incentives to honor the credit contract and absconds with the loan.

[32] Conditions below arise from a *minimax* problem. See proof of lemma 1 in FH for more details.

who have restricted access to credit and operate at an inefficient scale. Because in this range the marginal return to capital is larger than the marginal cost of debt, those agents will decide to ask for their maximum allowable loan given by

$$\Psi(a, d, l|\varphi_0, \theta_0) = 0, \tag{A.6}$$

where labor $l \equiv l(a|\varphi_0, \theta_0)$ satisfies,

$$p(1-s)f_l(a+d, (1-s)l) = \bar{w}(\varphi_0, \theta_0). \tag{A.7}$$

A.2 Occupational choice

In section 3.3, I define $\hat{a}_0$ as critical wealth level from which agents prefer to form a firm instead of becoming workers. Formally, this threshold is defined as follows,

$$\hat{a}_0 \equiv \inf_{\{a\}}\{U^e(a, d(a), l(a)) - u^w(a)\} \geq 0$$

Note that different arrangements could arise in the model as function of $\underline{a}_0$ and $\hat{a}_0$. Figure 19 illustrates these features. Panel a) shows the case in which $\underline{a}_0 > \hat{a}_0$. All agents with $a < \hat{a}_0$ become workers and those with $a \geq \underline{a}_0$ become entrepreneurs. Those agents with $a \in (\hat{a}_0, \underline{a}_0)$ may either become workers or invest their little wealth in a firm (micro-entrepreneurs). In the paper I study the case in which all agents with $a < \underline{a}_0$ become workers. Panel b) presents a case in which some agents that can access the credit market prefer to become workers, $a \in [\underline{a}_0, \hat{a}_0)$. In FH we show that the properties of the model are preserved under the cases that are not studied in this paper.

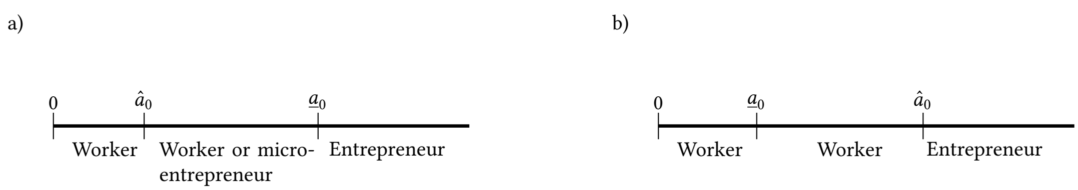

Figure 19: Occupational choice.

A.3 Measuring workers' welfare

This section shows that $U^w(l)$ is an appropriate measure of welfare for the group of workers matched to a firm hiring labor l.

Recall the labor market equilibrium condition,

$$l_s \cdot G(\underline{a}) = \int_{\underline{a}}^{a_M} l \, \partial G(a),$$

multiply by the expected wage $\bar{w}$ and subtract $\varsigma(l_s)G(\underline{a})$ on both sides,

$$\begin{aligned}
\underbrace{[\bar{w} \cdot l_s - \varsigma(l_s)]}_{=u^w(l_s)} G(\underline{a}) &= \left(\int_{\underline{a}}^{a_M} (\bar{w} \cdot l) \, \partial G(a) \right) - \varsigma(l_s)G(\underline{a}), \\
\Rightarrow u^w(l_s) \cdot G(\underline{a}) &= \left(\int_{\underline{a}}^{a_M} (\bar{w} \cdot l) \, \partial G(a) \right) - \left(\varsigma(l_s) \int_{\underline{a}}^{a_M} \frac{l}{l_s} \, \partial G(a) \right), \\
\Rightarrow u^w(l_s) \cdot G(\underline{a}) &= \int_{\underline{a}}^{a_M} U^w(l) \, \partial G(a). \qquad \text{(A.8)}
\end{aligned}$$

where in the second line I used the labor market equilibrium condition. Expression (A.8) shows how the aggregate workers' welfare in the economy, $u^w(l_s) \cdot G(\underline{a})$ is distributed across firms. It turns out that $U^w(l) = \bar{w}l - \frac{l}{l_s}\varsigma(l_s)$ is the correct measure for workers' welfare in firm l.

A.4 Individual workers' welfare under S-shaped EPLs

In this section I show how to obtain individual workers' welfare under an S-shaped policy characterized by the size threshold a^x. Define $u_0^w \equiv u^w(l_s(x_0))$ and $u_1^w \equiv u^w(l_s(x_1))$, where $l_s(x)$ is the individual labor supply given by (3.6). Since workers are randomly matched to firms, the expected utility of an individual worker, $\mathbb{E}u^w$ is given by

$$\mathbb{E}u^w = \frac{m^0}{m^0 + m^1} u_0^w + \frac{m^1}{m^0 + m^1} u_1^w, \qquad \text{(A.9)}$$

where m^0 and m^1 are the masses of workers that supply $l_s(x_0)$ and $l_s(x_1)$, respectively, as defined by conditions (5.2) and (5.3). Since the total mass of workers if given by $G(\underline{a}_0)$, the total workers' welfare in the economy, $\bar{U}^w$ is given by

$$\begin{aligned}
\bar{U}^w &= \left[\frac{m^0}{m^0 + m^1} u_0^w + \frac{m^1}{m^0 + m^1} u_1^w \right] \cdot G(\underline{a}_0) \\
&= m_0 u_0^w + m_1 u_1^w, \qquad \text{(A.10)}
\end{aligned}$$

where in the second line I have used condition (5.4). Based on equation (A.8) the following condition must hold

$$m_0 u_0^w + m_1 u_1^w = \int_{\underline{a}_0}^{a^x} U^w(a|x_0) \partial G + \int_{a^x}^{a_M} U^w(a|x_1) \partial G. \qquad \text{(A.11)}$$

Thus, the government's problem can be written in terms of any of these two measures. In this paper, I use the expression on the right-hand side because of two reasons: i) it allows to obtain proposition 3 and simplify the government's problem, and ii) it allows to characterize the political preferences of the different 'groups of workers', which admits a more intuitive interpretation of the results.

B Appendix: Main Proofs

Proposition 1 *Consider the initial labor regulation,* $\mathcal{P}_0 : [\underline{a}_0, a_M] \rightarrow \{\varphi_0, \theta_0\}$ *, then:*

1. *All entrepreneurs are worse off after a marginal increase of* φ *or* θ.
2. *This negative effect is strictly decreasing if* $a \in [\underline{a}_0, \bar{a}_0)$ *and remains constant after* $a \geq \bar{a}_0$.

Proof:

To simplify calculations, define $x = \{\varphi, \theta\}$. Differentiation of U^e in terms of x gives,

$$\frac{\partial U^e}{\partial x} = [p f_k - (1 + r^*)] \frac{\partial d}{\partial x} - \frac{\partial \bar{w}(\varphi, \theta)}{\partial x} l. \tag{B.1}$$

Define $\bar{w}_\varphi \equiv \frac{\partial \bar{w}(\varphi,\theta)}{\partial \varphi} = \frac{\partial w}{\partial \varphi}[p((1-s)+s\varphi)+(1-p)\theta] + psw$ and $\bar{w}_\theta \equiv \frac{\partial \bar{w}(\varphi,\theta)}{\partial \theta} = \frac{\partial w}{\partial \theta}[p((1-s)+s\varphi)+(1-p)\theta] + (1-p)w$. Additionally, use that $\frac{\partial d}{\partial x} = \frac{l \cdot \bar{w}_x}{f_k - (1+\underline{r})}$ in (B.1),

$$\frac{\partial U^e}{\partial x} = l \cdot \bar{w}_x \left[\frac{p f_k - (1 + r^*)}{p f_k - (1 + \underline{r})} - 1 \right] = \phi \bar{w}_x \frac{l}{p f_k - (1 + \underline{r})} < 0. \tag{B.2}$$

Thus, the effect of increased $x = \{\varphi, \theta\}$ on entrepreneurs' utility is negative. In particular, $\lim_{a \to \underline{a}_0^+} \frac{\partial U^e(a)}{\partial x} = -\infty$ and $\frac{\partial U^e(\bar{a}_0)}{\partial x} = -l^* \bar{w}_x$.

In order to conclude that this negative effect becomes weaker as a increases, all is left to show is that $\frac{\partial}{\partial a}\left(\frac{\partial U^e}{\partial x}\right) > 0$. Differentiate (B.2) with respect to a,

$$\frac{\partial}{\partial a}\left(\frac{\partial U^e}{\partial x}\right) = \frac{\phi \bar{w}_x}{(p f_k - (1 + \underline{r}))^2} \left[\frac{\partial l}{\partial a}(p f_k - (1 + \underline{r})) - l \frac{\partial}{\partial a}(p f_k) \right].$$

Note that,

$$\frac{\partial}{\partial a}(f_k) = \left(f_{kk} - \frac{f_{kl}^2}{f_{ll}} \right) \left(1 + \frac{\partial d}{\partial a} \right) = -\frac{\alpha f}{(1-\beta)k^2}(1 - \alpha - \beta) \left(1 + \frac{\partial d}{\partial a} \right) < 0, \tag{B.3}$$

then,

$$\begin{aligned}
\frac{\partial}{\partial a}\left(\frac{\partial U^e}{\partial x}\right) &= \frac{\phi \bar{w}_x}{(p f_k - (1 + \underline{r}))^2} \left(1 + \frac{\partial d}{\partial a} \right) \left[-\frac{f_{kl}}{(1-s) f_{ll}} (p f_k - (1 + \underline{r})) + l \frac{p \alpha f}{(1-\beta)k^2}(1 - \alpha - \beta) \right], \\
&= \frac{\phi \bar{w}_x}{(p f_k - (1 + \underline{r}))^2} \left(1 + \frac{\partial d}{\partial a} \right) \left[\frac{\alpha l}{k(1-s)^2(1-\beta)} (p f_k - (1 + \underline{r})) + l \frac{p f_k}{(1-\beta)k}(1 - \alpha - \beta) \right], \\
&= \underbrace{\frac{\phi l \bar{w}_x}{(1-s)^2(1-\beta)k(p f_k - (1 + \underline{r}))^2} \left(1 + \frac{\partial d}{\partial a} \right)}_{>0} \left[\alpha(p f_k - (1 + \underline{r})) + p f_k (1-s)^2 (1 - \alpha - \beta) \right].
\end{aligned}$$

Denote the term in brackets by h and notice that,

$$h \equiv \alpha(pf_k - (1+\underline{r})) + pf_k(1-s)^2(1-\alpha-\beta) > -\alpha\phi + (1+r^*)(1-s)^2(1-\alpha-\beta) > 0,$$

where the first inequality comes from $pf_k \in [1+r^*, 1+\underline{r}]$ and the second one uses assumption 1. Therefore, $\frac{\partial}{\partial a}\left(\frac{\partial U_e}{\partial x}\right) > 0, x \in \{\varphi, \theta\}$ and smaller firms are more adversely affected by an improvement in EPLs measured by φ or θ, which concludes the proof. ■

Proposition 2 *Consider the initial labor regulation, $\mathcal{P}_0 : [\underline{a}_0, a_M] \rightarrow \{\varphi_0, \theta_0\}$ and suppose a marginal increase of φ or θ. Then, there are cutoffs $\tilde{a}_0^{\varphi} \in (\underline{a}_0, \overline{a}_0)$ and $\tilde{a}_0^{\theta} \in (\underline{a}_0, \overline{a}_0)$ given by*

$$\frac{\partial U^w(\tilde{a}_0^x|\mathcal{P}_0)}{\partial x} = 0, \quad x \in \{\varphi, \theta\}$$

such that,

1. *Workers' welfare in firms with $a \in [\underline{a}_0, \tilde{a}_0^x)$ decreases.*
2. *Workers' welfare in firms with $a > \tilde{a}_0^x$ increases.*
3. *This marginal effect is strictly increasing in $a \in [\underline{a}_0, \overline{a}_0)$ and remains constant after $a \geq \overline{a}_0$.*

Proof: Differentiating condition (3.4) with respect to $x = \{\varphi, \theta\}$:

$$\begin{aligned}
\frac{\partial U^w(a|\varphi,\theta)}{\partial x} &= \bar{w}_x l + \frac{\partial l}{\partial x}\bar{w}(\varphi,\theta) - \frac{\left[\frac{\partial l}{\partial x}\varsigma(l_s) + l\varsigma'(l_s)\frac{\partial l_s}{\partial x}\right] l_s - l\varsigma(l_s)\frac{\partial l_s}{\partial x}}{(l_s)^2}, \\
&= \bar{w}_x l + \frac{\partial l}{\partial x}\left(\underbrace{\bar{w}(\varphi,\theta)}_{=\varsigma'(l_s)} - \frac{\varsigma(l_s)}{l_s}\right) - \frac{l}{l_s}\frac{\partial l_s}{\partial x}\left(\varsigma'(l_s) - \frac{\varsigma(l_s)}{l_s}\right), \\
&= \bar{w}_x \cdot l \underbrace{\left[1 - \frac{1}{\varsigma''(l_s)\cdot l_s}\left(\varsigma'(l_s) - \frac{\varsigma(l_s)}{l_s}\right)\right]}_{=(\gamma-1)/\gamma>0} + \underbrace{\frac{\partial l}{\partial x}}_{<0}\underbrace{\left(\varsigma'(l_s) - \frac{\varsigma(l_s)}{l_s}\right)}_{=(\gamma-1)l_s^{\gamma-1}>0},
\end{aligned} \tag{B.4}$$

where the last equality uses that $\frac{\partial l_s}{\partial x} = \frac{\bar{w}_x}{\varsigma''(l_s)} > 0$. Note that the sign of $\frac{\partial U^w(a|\varphi,\theta)}{\partial x}$ is ambiguous and depends on a. For a firm which is operating close enough to $\underline{a}_0$, $\lim_{a\to\underline{a}_0^+}\frac{\partial U^w(a|\varphi,\theta)}{\partial x} = -\infty$ (since $\lim_{a\to\underline{a}_0^+}\frac{\partial d}{\partial x} = -\infty$ and so, $\lim_{a\to\underline{a}_0^+}\frac{\partial l}{\partial x} = -\infty$). Thus, at least in a neighborhood of $\underline{a}_0$ workers are worse off. In addition, note that the labor market must satisfy the welfare equilibrium condition,

$$\int_{\underline{a}}^{a_M} u^w(\varphi,\theta)\partial G = \int_{\underline{a}}^{a_M} U_w(a|\varphi,\theta)\partial G, \tag{B.5}$$

Differentiate in terms of $x \in \{\varphi, \theta\}$ and evaluate at $\mathcal{P}_0$,

$$\underbrace{\frac{\partial u^w(\varphi_0, \theta_0)}{\partial x} G(\underline{a}_0) + u^w(\varphi_0, \theta_0) g(\underline{a}_0) \frac{\partial \underline{a}_0}{\partial x}}_{>0} = \int_{\underline{a}_0}^{a_M} \frac{\partial U_w(a|\varphi_0, \theta_0)}{\partial x} \partial G - U_w(a|\varphi_0, \theta_0) g(\underline{a}_0) \frac{\partial \underline{a}_0}{\partial x}, \tag{B.6}$$

where I have used that $\frac{\partial u^w(\varphi_0,\theta_0)}{\partial x} > 0$ and $\frac{\partial \underline{a}_0}{\partial x} > 0$.

Using the fact that $\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x} < 0$ in some neighborhood of $\underline{a}_0$ and that the second term of the right-hand side is also negative, it follows that $\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x}$ must be positive in some range (otherwise condition (B.6) is violated). If $\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x}$ is strictly increasing in a then there exist some threshold $\tilde{a}_0^x \equiv \tilde{a}^x(\mathcal{P}_0) \in (\underline{a}_0, \bar{a}_0)$ defined by

$$\frac{\partial U^w(\tilde{a}_0^x|\mathcal{P}_0)}{\partial x} = 0, \quad x \in \{\varphi, \theta\},$$

such that $\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x} < 0$ if $a \in [\underline{a}_0, \tilde{a}_0^x)$ and $\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x} > 0$ if $a > \tilde{a}_0^x$.

All is left to show is that $\frac{\partial}{\partial a}\left(\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x}\right) > 0$. Differentiation of $\frac{\partial U^w(a|\varphi,\theta)}{\partial x}$ with respect to a leads to:

$$\frac{\partial}{\partial a}\left(\frac{\partial U^w(a|\varphi, \theta)}{\partial x}\right) = \underbrace{\bar{w}_x \cdot \frac{\partial l}{\partial a}}_{>0} \underbrace{\left[1 - \frac{1}{\varsigma''(l_s) \cdot l_s}\left(\varsigma'(l_s) - \frac{\varsigma(l_s)}{l_s}\right)\right]}_{>0} + \frac{\partial}{\partial a}\left(\frac{\partial l}{\partial x}\right) \underbrace{\left(\varsigma'(l_s) - \frac{\varsigma(l_s)}{l_s}\right)}_{>0}.$$

Observe that the sign of $\frac{\partial}{\partial a}\left(\frac{\partial U^w(a|\varphi,\theta)}{\partial x}\right)$ depends on $\frac{\partial}{\partial a}\left(\frac{\partial l}{\partial x}\right)$. In what follows I show

From the FOC of labor (A.7),

$$\begin{aligned}
p(1-s)\left(f_{lk}\frac{\partial d}{\partial \theta} + (1-s) f_{ll}\frac{\partial l}{\partial \theta}\right) &= \bar{w}_x, \\
\Rightarrow \frac{\partial l}{\partial x} = \frac{\frac{\bar{w}_x}{p(1-s)} - f_{lk}\frac{\partial d}{\partial \theta}}{(1-s) f_{ll}} &= \frac{\bar{w}_x}{1-s}\left(\frac{1}{p(1-s) f_{ll}} - \frac{l f_{kl}}{f_{ll}(f_k - (1+\underline{r}))}\right) \\
&= \frac{\bar{w}_x}{1-s}\left(\frac{1}{p(1-s) f_{ll}} - \frac{\beta(1-s) f_k}{f_{ll}(p f_k - (1+\underline{r}))}\right),
\end{aligned} \tag{B.7}$$

where the last equality follows from $f_{kl} = \frac{\alpha(1-s)\beta f}{kl} = \frac{\beta(1-s)f_k}{l}$. Differentiation of (B.7) leads to:

$$\frac{\partial}{\partial a}\left(\frac{\partial l}{\partial x}\right) = \frac{\bar{w}_x}{1-s}\Bigg[- \frac{\frac{\partial}{\partial a}(f_{ll})}{p(1-s)f_{ll}^2} - \beta(1-s)\frac{\frac{\partial}{\partial a}(f_k)(pf_k-(1+\underline{r}))f_{ll}}{(pf_k-(1+\underline{r}))^2 f_{ll}^2} + \beta(1-s)f_k\frac{\big(p\frac{\partial}{\partial a}(f_k)f_{ll} + (pf_k-(1+\underline{r}))\frac{\partial}{\partial a}(f_{ll}}{(pf_k-(1+\underline{r}))^2 f_{ll}^2}$$

$$= \frac{\bar{w}_x}{1-s}\left(\frac{\partial}{\partial a}(f_{ll})\left[\frac{\beta(1-s)f_k(pf_k-(1+\underline{r}))}{(pf_k-(1+\underline{r}))^2 f_{ll}^2} - \frac{1}{p(1-s)f_{ll}^2}\right] + \beta(1-s)\frac{\partial}{\partial a}(f_k)\frac{f_{ll}(1+\underline{r})}{(pf_k-(1+\underline{r}))^2 f_{ll}^2}\right)$$

$$= \underbrace{\frac{\bar{w}_x}{p(1-s)^2(pf_k-(1+\underline{r}))^2 f_{ll}^2}}_{\equiv h_x>0}\Bigg[\frac{\partial}{\partial a}(f_{ll})\cdot[\beta(1-s)^2pf_k(pf_k-(1+\underline{r}))-(pf_k-(1+\underline{r}))^2]$$

$$+ \beta(1-s)^2p\frac{\partial}{\partial a}(f_k)\cdot f_{ll}(1+\underline{r})\Bigg]. \tag{B.8}$$

Notice that,

$$\frac{\partial}{\partial a}(f_{ll}) = f_{llk}\left(1+\frac{\partial d}{\partial a}\right) + f_{lll}(1-s)\frac{\partial l}{\partial a} = \left(f_{llk} - \frac{f_{kl}\cdot f_{lll}}{f_{ll}}\right)\left(1+\frac{\partial d}{\partial a}\right) = \frac{\alpha\beta(1-s)^2 f}{kl^2}\left(1+\frac{\partial d}{\partial a}\right) > 0. \tag{B.9}$$

Defining $\tilde{h}_x \equiv h_x\left(1+\frac{\partial d}{\partial a}\right)$ and replacing (B.9) and (B.3) in (B.8) gives:

$$\frac{\partial}{\partial a}\left(\frac{\partial l}{\partial \theta}\right) = \tilde{h}_x\Bigg[\frac{\alpha\beta(1-s)^2 f}{kl^2}\cdot[\beta(1-s)^2pf_k(pf_k-(1+\underline{r}))-(pf_k-(1+\underline{r}))^2]$$

$$- \beta(1-s)^2p\frac{\alpha f}{(1-\beta)k^2}(1-\alpha-\beta)\cdot f_{ll}(1+\underline{r})\Bigg],$$

$$= \tilde{h}_x\frac{f_{ll}}{k}\Bigg[\frac{\alpha}{(\beta-1)}\cdot[\beta(1-s)^2pf_k(pf_k-(1+\underline{r}))-(pf_k-(1+\underline{r}))^2]$$

$$- \beta(1-s)p\frac{f_k}{1-\beta}(1-\alpha-\beta)\cdot(1+\underline{r})\Bigg],$$

$$= \underbrace{-(1-\beta)^{-1}\tilde{h}_x\frac{f_{ll}}{k}}_{>0}\Bigg[\alpha[\beta(1-s)^2pf_k(pf_k-(1+\underline{r}))-(pf_k-(1+\underline{r}))^2]$$

$$+ \beta(1-s)^2pf_k(1-\alpha-\beta)(1+\underline{r})\Bigg].$$

The sign of this expression is determined by the sign of the term in brackets which we denote by q:

$$\begin{aligned} q &\equiv \alpha[\beta(1-s)^2pf_k(pf_k-(1+\underline{r}))-(pf_k-(1+\underline{r}))^2] + \beta(1-s)^2pf_k(1-\alpha-\beta)(1+\underline{r}), \\ &= \alpha(pf_k-(1+\underline{r}))[\beta(1-s)^2pf_k-(pf_k-(1+\underline{r}))] + \beta(1-s)^2pf_k(1-\alpha-\beta)(1+\underline{r}), \\ &= -\alpha(pf_k-(1+\underline{r}))(pf_k(1-\beta(1-s)^2)-(1+\underline{r})) + \beta(1-s)^2pf_k(1-\alpha-\beta)(1+\underline{r}). \end{aligned}$$

Recall that $pf_k \in [1+r^*, 1+\underline{r}]$, then $pf_k - (1+\underline{r}) \in [-\phi, 0]$ and $pf_k(1-\beta(1-s)^s) - (1+\underline{r}) \in [-(\beta(1-s)^2(1+r^*)+\phi), -\beta(1-s)^2(1+r^*+\phi)]$. Using these properties and assumption 1,

$$\begin{aligned} q &\geq -\alpha\phi(\beta(1-s)^2(1+r^*)+\phi) + \beta(1-s)^2(1+r^*)(1-\alpha-\beta)(1+r^*+\phi), \\ &> -\alpha\phi(\beta(1-s)^2(1+r^*)+\phi) + \beta(1-s)^2(1+r^*)(1-\alpha-\beta)(\beta(1-s)^2(1+r^*)+\phi), \\ &> (\beta(1-s)^2(1+r^*)+\phi)\big[-\alpha\phi + \beta(1-s)^2(1+r^*)(1-\alpha-\beta)\big] > 0, \end{aligned}$$

which implies that $\frac{\partial}{\partial a}\left(\frac{\partial l}{\partial x}\right) > 0$. Thus, $\frac{\partial}{\partial a}\left(\frac{\partial U^w(a|\varphi_0,\theta_0)}{\partial x}\right) > 0$, which leads to the result of the proposition. ■

Proposition 3 *Any labor regulation policy, $\mathcal{P}$ that solves* (3.14) *satisfies monotonicity at each component,*

$$\mathcal{P}^x(a) : \mathcal{P}^x(a') \leq \mathcal{P}^x(a'') \quad \forall a' < a'', x \in \{\varphi, \theta\}.$$

Moreover, there are size thresholds $a^\varphi \in [\underline{a}_0, a_M]$ and $a^\theta \in [\underline{a}_0, a_M]$ such that:

$$\mathcal{P}^x(a) = \begin{cases} x_0 \ \textit{if}\, a < a^x, \\ x_1 \ \textit{if}\, a \geq a^x. \end{cases}$$

Proof: By contradiction, suppose that there is some solution to problem (3.14), $\mathcal{P}^x(a)$ such that it violates monotonicity in some non-zero measure set $\mathcal{A} \in \mathcal{B}([\underline{a}_0, a_M])$ and for which monotonicity holds in $[\underline{a}_0, a_M] - \{\mathcal{A}\}$.

Let x_i, with $i \in \{0,1\}$ be defined as,

$$x_i = \begin{cases} \varphi_i & \text{if } x = \varphi, \\ \theta_i & \text{if } x = \theta. \end{cases}$$

Assume that $\mathcal{A}$ satisfies,

$\mathcal{A} : \mathcal{A} = \mathcal{A}_0 \bigcup \mathcal{A}_1$ with $\mathcal{A}_0 \bigcap \mathcal{A}_1 = \emptyset$ and $a' \in \mathcal{A}_0, a'' \in \mathcal{A}_1 \Rightarrow a' < a''$,

and define,

$$\mathcal{P}^x(a) : \mathcal{P}^x(a') > \mathcal{P}^x(a''), a' \in \mathcal{A}_0, a'' \in \mathcal{A}_1.$$

This last condition is equivalent to $\mathcal{P}^x(\mathcal{A}_0) > \mathcal{P}^x(\mathcal{A}_1) \Leftrightarrow \mathcal{P}^x(\mathcal{A}_0) = x_1$ and $\mathcal{P}^x(\mathcal{A}_1) = x_0$.

Further, define $m^e_G(x_0|\mathcal{P}, \mathcal{A})$ and $m^e_G(x_1|\mathcal{P}, \mathcal{A})$ as the masses of entrepreneurs in the set $\mathcal{A}$ that operate under x_0 and x_1 when $\mathcal{P}$ is implemented,

$$m^e_G(x_i|\mathcal{P}, \mathcal{A}) = \int_{a\in\mathcal{A}} \mathbf{1}[\mathcal{P}^x(a) = x_i]\partial G, \ \ i \in \{0,1\} \tag{B.10}$$

Consider the alternative labor regulation policy, $\mathcal{P}^{x'}$ such that,

$$\mathcal{P}^{x'}(a) = \begin{cases} \mathcal{P}^x(a) & \text{if } a \in [\underline{a}_0, a_M] - \{\mathcal{A}\}, \\ \{\mathcal{P}^{x'}(a) : \mathcal{P}^{x'}(\tilde{\mathcal{A}}_0) < \mathcal{P}^{x'}(\tilde{\mathcal{A}}_1)\} & \text{if } a \in \mathcal{A} = \tilde{\mathcal{A}}_0 \bigcup \tilde{\mathcal{A}}_1, \end{cases}$$

where,
$\{\mathcal{A} : \mathcal{A} = \tilde{\mathcal{A}}_0 \bigcup \tilde{\mathcal{A}}_1$ with $\tilde{\mathcal{A}}_0 \bigcap \tilde{\mathcal{A}}_1 = \emptyset$ and $a' \in \tilde{\mathcal{A}}_0, a'' \in \tilde{\mathcal{A}}_1 \Rightarrow a' < a''\}$,
and
$\{\tilde{\mathcal{A}}_0, \tilde{\mathcal{A}}_1 : m^e_G(x_0|\mathcal{P}^{x'}, \mathcal{A}) = m^e_G(x_0|\mathcal{P}^x, \mathcal{A})$ and $m^e_G(x_1|\mathcal{P}^{x'}, \mathcal{A}) = m^e_G(x_1|\mathcal{P}^x, \mathcal{A})\}$.

Observe that $\mathcal{P}^{x'}(\tilde{\mathcal{A}}_0) = x_0$ and $\mathcal{P}^{x'}(\tilde{\mathcal{A}}_1) = x_1$. Thus, $\mathcal{P}^{x'}$ satisfies monotonicity in $\mathcal{A}$. Moreover, it reverts and preserves the masses of entrepreneurs operating under x_0 and x_1 that arise from $\mathcal{P}^x$. From proposition 1, $\frac{\partial}{\partial a}\left(\frac{\partial U^e}{\partial x}\right) > 0$, thus the aggregate welfare of entrepreneurs is higher under $\mathcal{P}^{x'}$. Additionally, proposition 2 shows that $\frac{\partial}{\partial a}\left(\frac{\partial U^w}{\partial x}\right) > 0$, hence workers' welfare is also larger under $\mathcal{P}^{x'}$. Therefore, $\mathcal{P}^x$ cannot solve problem (3.14).

Nevertheless, observe that $\mathcal{P}^{x'}$ may not satisfy monotonicity in $[\underline{a}_0, a_M]$. For instance, if $\mathcal{P}^x$ was such that $\mathcal{P}^x(a) = x_1, \forall a$. But since $\mathcal{A}$ was chosen arbitrarily, the argument can be repeated iteratively to discard any solution for which monotonicity does not hold in some non-zero measure set. Hence, the solution to the government's problem must satisfy monotonicity in both components.[33] Thus, by monotonicity of $\mathcal{P}^x(a)$ there is some $a^x \in [\underline{a}_0, a_M]$ such that $\mathcal{P}^x(a) = x_0$ if $a < a^x$ and $\mathcal{P}^x(a) = x_1$ if $a \geq a^x$. ■

Proposition 4 *The equilibrium size threshold, a^x_{pe} under sticky wages is as follows:*

1. *If $\lambda \leq \frac{1}{2+1/(\gamma-2)}$, then $a^x_{pe} = a_M$.*

2. *If $\lambda > \frac{1}{2-1/\gamma}$, then $a^x_{pe} \in [\tilde{a}^x_0, \overline{a}_0)$ satisfies,*

$$\lambda \frac{\partial U^w(a^x_{pe}|x_0)}{\partial x} = -(1-\lambda)\frac{\partial U^e(a^x_{pe}|x_0)}{\partial x}.$$

In particular, if $\lambda = 1$, then $a^x_{pe} = \tilde{a}^x_0$ and $a^x_{pe} > \tilde{a}^x_0$ if $\lambda < 1$.

Proof: The FOC of the government's problem is as follows,

$$\lambda[U^w(l(a^x_{gp}|x_0)) - U^w(l(a^x_{gp}|x_1))]g(a^x_{gp}) + (1-\lambda)[U^e(k(a^x_{gp}), l(a^x_{gp})|x_0) - U^e(k(a^x_{gp}), l(a^x_{gp})|x_1)]g(a^x_{gp}) = 0.$$

[33] Notice that the resulting $\mathcal{P}^{x'}$ is not necessarily the solution. It is an arbitrary labor regulation policy that satisfies monotonicity and that dominates any $\mathcal{P}^x$ that violates monotonicity in some non-zero measure set.

Rearranging terms,

$$(2\lambda-1)[\bar{w}(x_0)l(a^x_{gp}|x_0)-\bar{w}(x_1)l(a^x_{gp}|x_1)]-\lambda\left[\frac{l(a^x_{gp}|x_0)}{l_s(a^x_{gp}|x_0)}\varsigma(l_s(a^x_{gp}|x_0))-\frac{l(a^x_{gp}|x_1)}{l_s(a^x_{gp}|x_1)}\varsigma(l_s(a^x_{gp}|x_1))\right]$$
$$+(1-\lambda)[\tilde{f}(a^x_{gp}|x_0)-\tilde{f}(a^x_{gp}|x_1)]=0,$$

where I have defined,

$$\tilde{f}(a|x)\equiv pf(k(a|x),l(a|x))+(1-p)\eta k(a|x)-(1+\rho)d(a|x)-F, \tag{B.11}$$

which corresponds to expected firm's output net of credit and operation costs. Define now a measure of 'weighted worker's welfare' as follows,

$$\hat{U}^w(a|x)=(2\lambda-1)\bar{w}(x)l(a|x)-\lambda\frac{l(a|x)}{l_s(x)}\varsigma(l_s(x)). \tag{B.12}$$

Then the FOC reads as,

$$\hat{U}(a^x_{gp}|x_0)-\hat{U}(a^x_{gp}|x_1)=\tilde{f}(a^x_{gp}|x_1)-\tilde{f}(a^x_{gp}|x_1)$$

Divide both sides of previous expression by Δ and take $\lim_{\Delta\to 0}(\cdot)$ to obtain,[34]

$$\frac{\partial\hat{U}(a^x_{gp}|x_0)}{\partial x}=-(1-\lambda)\frac{\partial\tilde{f}(a^x_{gp}|x_0)}{\partial x}. \tag{B.13}$$

Analogously to expression (B.4), differentiation of (B.12) in terms of $x\in\{\varphi,\theta\}$ leads to,

$$\frac{\partial}{\partial x}\left(\hat{U}^w(a|\varphi,\theta)\right)=\bar{w}_x\cdot l\left[(2\lambda-1)-\frac{1}{\varsigma''(l_s)\cdot l_s}\left((2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}\right)\right]+\underbrace{\frac{\partial l}{\partial x}}_{<0}\left((2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}\right) \tag{B.14}$$

In what follows expression (B.14) is used to characterize the solution to (B.13). Two cases are studied: i) $\lambda\leq\frac{1}{2+1/\gamma-2}$ and ii) $\lambda>\frac{1}{2-1/\gamma}$. When $\lambda\in\left[\frac{1}{2+1/\gamma-2},\frac{1}{2-1/\gamma}\right]$ there may exist multiple solutions.

Case 1: $\lambda\leq\frac{1}{2+1/(\gamma-2)}$

Note that in this case,

$$(2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}=[(2\lambda-1)\gamma-\lambda](l_s)^{\gamma-1}<0,$$

[34]Observe that this expression is analogous to (5.6). As will be clear later, this alternative form is useful for the study of the solution to the government's problem.

and

$$(2\lambda-1)-\frac{1}{\varsigma''(l_s)\cdot l_s}\left((2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}\right)=\frac{(2\lambda-1)\gamma(\gamma-2)+\lambda}{\gamma(\gamma-1)}<\frac{\lambda(2(\gamma-2)+1)+\gamma-2}{\gamma(\gamma-1)}<0.$$

Proceeding as in proposition 2, differentiation of (B.14) in terms of a leads to,

$$\frac{\partial}{\partial a}\left(\frac{\partial\hat{U}^w(a|x_0)}{\partial x}\right)=\underbrace{\bar{w}_x\cdot\frac{\partial l}{\partial a}}_{>0}\underbrace{\left[(2\lambda-1)-\frac{1}{\varsigma''(l_s)\cdot l_s}\left((2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}\right)\right]}_{<0}$$
$$+\underbrace{\frac{\partial}{\partial a}\left(\frac{\partial l}{\partial x}\right)}_{>0}\underbrace{\left((2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}\right)}_{<0}<0.$$

Hence, the left-hand side of (B.13) decreasing in a. Using a similar argument as the one used in proposition 2, $\lim_{a\to\underline{a}_0^+}\frac{\partial\hat{U}^w(a|x_0)}{\partial x}=+\infty$ and there is some cutoff $\hat{a}_0^x\in(\underline{a}_0,\overline{a}_0)$ given by

$$\frac{\partial\hat{U}(\hat{a}_0^x|x_0)}{\partial x}=0,$$

such that $\frac{\partial\hat{U}^w(a|x_0)}{\partial x}>0$ if $a<\hat{a}_0^x$ and $\frac{\partial\hat{U}^w(a|x_0)}{\partial x}<0$ if $a>\hat{a}_0^x$. Moreover, from proposition 1,

$$\frac{\partial}{\partial a}\left(-\frac{\partial\tilde{f}(a|x_0)}{\partial x}\right)<0.$$

Thus, the right-hand side of (B.13) is also decreasing in a. Additionally, $\lim_{a\to\underline{a}_0^+}-\frac{\partial\tilde{f}(a|x_0)}{\partial x}=+\infty$ and $-\frac{\partial\tilde{f}(a|x_0)}{\partial x}=0$ for $a\geq\overline{a}_0$. Since $\frac{\partial\hat{U}^w(\hat{a}_0^x|x_0)}{\partial x}=0$ and $\hat{a}_0^x<\overline{a}_0$, then $-\frac{\partial\tilde{f}(a|x_0)}{\partial x}$ is always above $\frac{\partial\hat{U}^w(a|x_0)}{\partial x}$. Thus, there is no solution for equation (B.13). Figure 23 in section E of this appendix illustrates condition (B.13) in terms of a_{gp}^x. The left-hand side is represented by the red solid line, while the blue dashed line depicts the right-hand side.

In conclusion, the solution of the government's problem is $\mathcal{P}_{gp}(a)=(\varphi_0,\theta_0)$, i.e. $a_{gp}^x=a_M$.

Case 2: $\lambda>\frac{1}{2-1/\gamma}$

Note that this condition is equivalent to $\gamma>\frac{\lambda}{2\lambda-1}$. Thus,

$$(2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}=[(2\lambda-1)\gamma-\lambda](l_s)^{\gamma-1}>0$$

and

$$(2\lambda-1)-\frac{1}{\varsigma''(l_s)\cdot l_s}\left((2\lambda-1)\varsigma'(l_s)-\lambda\frac{\varsigma(l_s)}{l_s}\right)=\frac{(2\lambda-1)\gamma(\gamma-2)+\lambda}{\gamma(\gamma-1)}>\frac{\lambda(\gamma-1)}{\gamma(\gamma-1)}=\frac{\lambda}{\gamma}>0.$$

Using the same argument used in proposition 2, $\frac{\partial}{\partial a}\left(\frac{\partial \hat{U}^w(a|x_0)}{\partial x}\right) > 0$. Moreover , $\lim_{a\to \underline{a}_0^+} \frac{\partial \hat{U}^w(a|x_0)}{\partial x} = -\infty$ and there is a cutoff $\hat{a}_0^x \in (\underline{a}_0, \overline{a}_0)$ such that $\frac{\partial \hat{U}^w(a|x_0)}{\partial x} < 0$ if $a < \hat{a}_0^x$ and $\frac{\partial \hat{U}^w(a|x_0)}{\partial x} > 0$ if $a > \hat{a}_0^x$.[35]

As in the previous case, figure 24 in section E illustrates equation (B.13) in terms of a_{gp}^x. There is a unique solution $a_{gp}^x \in (\hat{a}_0^x, \overline{a}_0)$ to equation (B.13). In particular, when $\lambda = 1$ the FOC reads as $\frac{\partial U^w(a_{gp}^x|x_0)}{\partial x} = 0$, which by proposition 2 is solved by $a_{gp}^x = \tilde{a}_0^x$. Otherwise, when $\lambda \in (\frac{1}{2-1/\gamma}, 1)$, $a_{gp}^x > \hat{a}_0^x > \tilde{a}_0^x$, as shown in the figure. ■

Lemma 1 *If $\lambda > \frac{1}{2-1/\gamma}$, the equilibrium size threshold, a_{gp}^x under sticky wages is strictly decreasing in λ.*

Proof: Differentiating (5.6) in terms of λ,

$$\frac{\partial U^w}{\partial x} + \lambda \cdot \frac{\partial^2 U^w}{\partial a_{gp}^x \partial x}\frac{\partial a_{gp}^x}{\partial \lambda} = \frac{\partial U^e}{\partial x} - (1-\lambda)\cdot \frac{\partial^2 U^e}{\partial a_{gp}^x \partial x}\frac{\partial a_{gp}^x}{\partial \lambda},$$

$$\Rightarrow \frac{\partial a_{gp}^x}{\partial \lambda} = \frac{\frac{\partial U^e}{\partial x} - \frac{\partial U^w}{\partial x}}{\lambda \frac{\partial^2 U^w}{\partial a_{gp}^x \partial x} + (1-\lambda)\frac{\partial^2 U^e}{\partial a_{gp}^x \partial x}}. \tag{B.15}$$

Note that from (5.6),

$$\lambda\left(\frac{\partial U^w}{\partial x} - \frac{\partial U^e}{\partial x}\right) = -\frac{\partial U^e}{\partial x} > 0,$$

thus, the numerator of (B.15) is negative. Finally, from propositions 1 and 2, the denominator is positive. Thus, $\frac{\partial a_{gp}^x}{\partial \lambda} < 0$, when $\lambda > \frac{1}{2-1/\gamma}$. ■

Lemma 2 *The equilibrium wage w is increasing in a^x. In particular, if $a^x = \underline{a}_0$, the change in w is such that $\frac{\partial \bar{w}}{\partial a^x} = 0$.*

Proof: Recall the labor market equilibrium conditions:

$$m^0 \cdot l_s(x_0) = \int_{\underline{a}}^{a^x} l(a|x_0)\partial G, \tag{B.16}$$

$$m^1 \cdot l_s(x_1) = \int_{a^x}^{a_M} l(a|x_1)\partial G, \tag{B.17}$$

$$m^0 + m^1 = G(\underline{a}). \tag{B.18}$$

[35] Since $\lambda > 2\lambda - 1$ when $\lambda \in (0, 1)$, then the cutoff at which $\frac{\partial \hat{U}^w}{\partial x} = 0$ is to the right of that at which $\frac{\partial U^w}{\partial x} = 0$.

Differentiation of conditions (B.16) to (B.18) in terms of a^x leads to,

$$\frac{\partial m^0}{\partial a^x} l_s^0 + m^0 \frac{\partial l_s^0}{\partial a^x} = \int_{\underline{a}}^{a^x} \frac{\partial l^0(a)}{\partial a^x} \partial G + l^0(a^x) g(a^x) - l^0(\underline{a}) g(\underline{a}) \frac{\partial \underline{a}}{\partial a^x}, \tag{B.19}$$

$$\frac{\partial m^1}{\partial a^x} l_s^1 + m_1 \frac{\partial l_s^1}{\partial a^x} = \int_{a^x}^{a_M} \frac{\partial l^1(a)}{\partial a^x} \partial G - l^1(a^x) g(a^x), \tag{B.20}$$

$$\frac{\partial m^1}{\partial a^x} = g(\underline{a}) \frac{\partial \underline{a}}{\partial a^x} - \frac{\partial m^0}{\partial a^x}, \tag{B.21}$$

where I have defined $l^0(a) \equiv l(a|x_0), l^1(a) \equiv l(a|x_1), l_s^0 \equiv l_s(x_0)$ and $l_s^1 \equiv l_s(x_1)$.

Combining (B.20) and (B.21),

$$\frac{\partial m^0}{\partial a^x} = \left(- \int_{a^x}^{a_M} \frac{\partial l^1(a)}{\partial a^x} \partial G + l^1(a^x) g(a^x) + l_s^1 g(\underline{a}) \frac{\partial \underline{a}}{\partial a^x} + m_1 \frac{\partial l_s^1}{\partial a^x} \right) \frac{1}{l_s^1}, \tag{B.22}$$

rearranging (B.19),

$$\frac{\partial m^0}{\partial a^x} = \left(\int_{\underline{a}}^{a^x} \frac{\partial l^0(a)}{\partial a^x} \partial G + l^0(a^x) g(a^x) - l^0(\underline{a}) g(\underline{a}) \frac{\partial \underline{a}}{\partial a^x} - m^0 \frac{\partial l_s^0}{\partial a^x} \right) \frac{1}{l_s^0}. \tag{B.23}$$

Equalizing conditions (B.22) and (B.23),

$$l_s^1 \int_{\underline{a}}^{a^x} \frac{\partial l^0(a)}{\partial a^x} \partial G + l_s^0 \int_{a^x}^{a_M} \frac{\partial l^1(a)}{\partial a^x} \partial G - l_s^1 (l^0(\underline{a}) + l_s^0) g(\underline{a}) \frac{\partial \underline{a}}{\partial a^x} - m^0 l_s^1 \frac{\partial l_s^0}{\partial a^x} - m^1 l_s^0 \frac{\partial l_s^1}{\partial a^x} = (l_s^0 l^1(a^x) - l_s^1 l^0(a^x)) g(a^x),$$

$$\Rightarrow \frac{\partial w}{\partial a^x} \left(l_s^1 \int_{\underline{a}}^{a^x} \underbrace{\frac{\partial l^0(a)}{\partial w}}_{<0} \partial G + l_s^0 \int_{a^x}^{a_M} \underbrace{\frac{\partial l^1(a)}{\partial w}}_{<0} \partial G - l_s^1 (l^0(\underline{a}) + l_s^0) g(\underline{a}) \underbrace{\frac{\partial \underline{a}}{\partial w}}_{>0} - m^0 l_s^1 \underbrace{\frac{\partial l_s^0}{\partial w}}_{>0} - m^1 l_s^0 \underbrace{\frac{\partial l_s^1}{\partial w}}_{>0} \right) = \underbrace{(l_s^0 l^1(a^x) - l_s^1 l^0(a^x))}_{<0} g(a^x).$$

This last condition implies that $\frac{\partial w}{\partial a^x} > 0$. Finally, suppose that $a^x \leq \underline{a}_0$, that is EPL increases from x_0 to x_1 for all firms. Recall the equilibrium labor market condition under a flat labor policy,

$$l_s G(\underline{a}) = \int_{\underline{a}}^{a_M} l(a) \partial G.$$

Differentiation in terms of $x = \{\varphi, \theta\}$ leads to,

$$\frac{\partial l_s}{\partial x} G(\underline{a}) + l_s g(\underline{a}) \frac{\partial \underline{a}}{\partial x} = \int_{\underline{a}}^{a_M} \frac{\partial l}{\partial x} \partial G,$$

$$\Rightarrow \frac{\partial \bar{w}}{\partial x} \underbrace{\left(\frac{\partial l_s}{\partial \bar{w}} G(\underline{a}) + l_s g(\underline{a}) \frac{\partial \underline{a}}{\partial \bar{w}} - \int_{\underline{a}}^{a_M} \frac{\partial l}{\partial \bar{w}} \partial G \right)}_{>0} = 0,$$

where I have used that $\frac{\partial l_s}{\partial x} = \frac{\partial \bar{w}}{\partial x}\frac{\partial l_s}{\partial \bar{w}}$, $\frac{\partial \underline{a}}{\partial x} = \frac{\partial \bar{w}}{\partial x}\frac{\partial \underline{a}}{\partial \bar{w}}$ and $\frac{\partial l}{\partial x} = \frac{\partial \bar{w}}{\partial x}\frac{\partial l}{\partial \bar{w}}$. In conclusion, $\frac{\partial \bar{w}}{\partial x} = 0$ if $a^x \leq \underline{a}_0$.[36] ■

Proposition 5

1. *$\bar{U}(a^x, \lambda)$ achieves a global maximum in $[\underline{a}_0, a_M]$ at some size threshold $a^x_{pe} \in (\underline{a}_0, a_M)$ characterized by*

$$a^x_{pe} = \sup_{a^x} \bar{U}(a^x, \lambda).$$

Suppose that $g(\cdot)$ satisfies $g' < 0$, then,

2. *$\bar{U}^e(a^x, \lambda)$ and $\bar{U}^w(a^x, \lambda)$ are strictly concave in a^x.*

3. *The equilibrium size threshold a^x_{pe} under flexible wages is the unique solution to,*

$$\lambda \frac{\partial \bar{U}^w(a^x_{pe}, \lambda)}{\partial a^x} = -(1-\lambda)\frac{\partial \bar{U}^e(a^x_{pe}, \lambda)}{\partial a^x}, \quad x \in \{\varphi, \theta\}.$$

4. *The equilibrium size threshold a^x_{pe} is decreasing in λ.*

Proof: Differentiation of equations (5.8) and (5.7) in terms of a^x leads to,

$$\begin{aligned}\frac{\partial \bar{U}^e(a^x)}{\partial a^x} &= \int_{\underline{a}_0}^{a^x} \frac{\partial U^e(a|x_0)}{\partial a^x}\partial G + \int_{a^x}^{a_M} \frac{\partial U^e(a|x_1)}{\partial a^x}\partial G + [U^e(a^x|x_0) - U^e(a^x|x_1)]g(a^x), \\ &= \frac{\partial w}{\partial a^x}\left[\int_{\underline{a}_0}^{a^x} \frac{\partial U^e(a|x_0)}{\partial w}\partial G + \int_{a^x}^{a_M} \frac{\partial U^e(a|x_1)}{\partial w}\partial G\right] + [U^e(a^x|x_0) - U^e(a^x|x_1)]g(a^x).\end{aligned} \tag{B.24}$$

$$\begin{aligned}\frac{\partial \bar{U}^w(a^x)}{\partial a^x} &= \int_{\underline{a}_0}^{a^x} \frac{\partial U^w(a|x_0)}{\partial a^x}\partial G + \int_{a^x}^{a_M} \frac{\partial U^w(a|x_1)}{\partial a^x}\partial G + [U^w(a^x|x_0) - U^w(a^x|x_1)]g(a^x), \\ &= \frac{\partial w}{\partial a^x}\left[\int_{\underline{a}_0}^{a^x} \frac{\partial U^w(a|x_0)}{\partial w}\partial G + \int_{a^x}^{a_M} \frac{\partial U^w(a|x_1)}{\partial w}\partial G\right] + [U^w(a^x|x_0) - U^w(a^x|x_1)]g(a^x).\end{aligned} \tag{B.25}$$

Proof of Item 1

I start by showing that $\bar{U}^e$ and $\bar{U}^w$ achieve a global maximum. First, recall that $\lim_{a \to \underline{a}_0} \frac{\partial U^w(a|x_0)}{\partial x} = -\infty$ and $\lim_{a \to \underline{a}_0} \frac{\partial U^e(a|x_0)}{\partial x} = -\infty$ (see the proofs of propositions 1 and 2). Therefore, $\lim_{a^x \to \underline{a}_0^+} \frac{\partial \bar{U}^w(a^x)}{\partial a^x} >$

[36]Note that the proof works even when $\underline{a}$ responds to a change in a^x. In particular, the result holds when the minimum wealth does not change (i.e $\frac{\partial \underline{a}}{\partial x} = 0$) and is given by $\underline{a}_0$.

0 and $\lim_{a^x \to \underline{a}_0^+} \frac{\partial U^e(a^x)}{\partial a^x} > 0$. Secondly, note that $\bar{U}^w(a^x)$ and $\bar{U}^e(a^x)$ are bounded in $[\underline{a}_0, a_M]$,

$$\bar{U}^e(a^x) < M^e \equiv U^e(a_M|x_0)[1 - G(\underline{a}_0)], \quad \forall a^x \in [\underline{a}_0, a_M],$$
$$\bar{U}^w(a^x) < M^w \equiv U^w(\bar{a}_0|x_1)[1 - G(\underline{a}_0)], \quad \forall a^x \in [\underline{a}_0, a_M],$$

where I have used the result of proposition 1 that $U^e(a|x)$ is increasing in a and decreasing in x. Also, proposition 2 states that $U^w(a|x)$ is weakly increasing in (a, x) and positive for $a \in [\tilde{a}_0^x, a_M]$. Thus, $\bar{U}^e(a^x)$ and $\bar{U}^w(a^x)$ are bounded by some finite positive numbers M^w and M^e, respectively.

In conclusion, $\bar{U}^e(a^x)$ and $\bar{U}^w(a^x)$ are continuous and bounded functions in $[\underline{a}_0, a_M]$ satisfying: i) $\bar{U}^e(\underline{a}_0) = \bar{U}^e(a_M) > 0$ and $\bar{U}^w(\underline{a}_0) = \bar{U}^w(a_M) > 0$,[37] ii) $\frac{\partial \bar{U}^e(\underline{a}_0^+)}{\partial a^x} > 0$ and $\frac{\partial \bar{U}^w(\underline{a}_0^+)}{\partial a^x} > 0$. Thus, $\bar{U}^e(a^x)$ and $\bar{U}^w(a^x)$ achieve a global maximum $\tilde{M}^e > \bar{U}^e(\underline{a}_0)$ and $\tilde{M}^w > \bar{U}^w(\underline{a}_0)$ given by

$$\tilde{M}^e = \sup_{a^x} \bar{U}^e(a^x), \quad x \in \{\varphi, \theta\},$$
$$\tilde{M}^w = \sup_{a^x} \bar{U}^w(a^x), \quad x \in \{\varphi, \theta\},$$

In consequence, $\bar{U} = \lambda \bar{U}^w + (1 - \lambda)\bar{U}^e$ achieves a global maximum. Moreover, properties i) and ii) imply that the global maximum is achieved at some $a_{gp}^x \in (\underline{a}_0, a_M)$. Thus, the equilibrium policy is S-shaped regardless of the value of λ.

Proof of Item 2

Differentiation of (B.24) and (B.25) in terms of a^x leads to,

$$\frac{\partial^2 \bar{U}^e}{\partial a^{x2}} = -2 \left[\frac{\partial U^e(a^x|x_1)}{\partial a^x} - \frac{\partial U^e(a^x|x_0)}{\partial a^x} \right] \cdot g(a^x) - [U^e(a^x|x_1) - U^e(a^x|x_0)] \cdot g'(a^x), \tag{B.26}$$
$$\frac{\partial^2 \bar{U}^w}{\partial a^{x2}} = -2 \left[\frac{\partial U^w(a^x|x_1)}{\partial a^x} - \frac{\partial U^w(a^x|x_0)}{\partial a^x} \right] \cdot g(a^x) - [U^w(a^x|x_1) - U^w(a^x|x_0)] \cdot g'(a^x). \tag{B.27}$$

Propositions 1 and 2 show that $\frac{\partial^2 U^e}{\partial a \partial x} > 0$ and $\frac{\partial^2 U^w}{\partial a \partial x} > 0$. Thus, the first terms of equations (B.26) and (B.27) are negative. Moreover, recall that $\frac{\partial U^e}{\partial x} < 0$. Thus, if $g' < 0$, then the second term of (B.26) is negative. Therefore, $\frac{\partial^2 \bar{U}^e}{\partial a^{x2}} < 0$ and $\bar{U}^e$ is strictly concave in a^x. Note however that the sign of $\frac{\partial U^w}{\partial x}$ depends on a^x. In particular, if $a^x > \tilde{a}_0^x$ then from proposition 2, $\frac{\partial U_w}{\partial x} > 0$ and the sign of (B.27) is ambiguous.

In order to find the sign of (B.27), I use the fact that the labor market satisfies the following welfare condition,

$$\bar{U}^w = u^w(x_0)m^0 + u^w(x_1)m^1.$$

[37]These properties come from the fact that having $a^x = \underline{a}_0$ or $a^x = a_M$ leads to the same expected wage $\bar{w}$ and thus, to the same equilibrium outcomes (see the last part of lemma 2)

Differentiating twice in terms of a^x gives,

$$\begin{aligned}\frac{\partial^2 \bar{U}^w}{\partial a^{x2}} &= -2\left[\frac{\partial u^w(x_1)}{\partial a^x} - \frac{\partial u^w(x_0)}{\partial a^x}\right]\frac{\partial m^0}{\partial a^x}, \\ &\quad - 2\underbrace{\frac{\partial w}{\partial a^x}}_{>0}\left[\frac{\partial u^w(x_1)}{\partial w} - \frac{\partial u^w(x_0)}{\partial w}\right]\underbrace{\frac{\partial m^0}{\partial a^x}}_{>0},\end{aligned} \tag{B.28}$$

where I have used that $\frac{\partial m_1}{\partial a^x} = -\frac{\partial m_0}{\partial a^x}$. For the term in square brackets note that,

$$\frac{\partial u^w}{\partial x} = \frac{\partial \bar{w}}{\partial x} l_s + \underbrace{(\bar{w} - \varsigma'(l_s))}_{=0}\frac{\partial l_s}{\partial x},$$

therefore,

$$\frac{\partial^2 u^w}{\partial w \partial x} = \underbrace{\frac{\partial^2 \bar{w}}{\partial w \partial x}}_{>0} l_s + \underbrace{\frac{\partial \bar{w}}{\partial x}\frac{\partial l_s}{\partial w}}_{>0} > 0,$$

In conclusion, (B.28) is negative and $\bar{U}^w$ is also strictly concave in a^x.

Proof of Item 3

Since both $\bar{U}^e$ and $\bar{U}^w$ are strictly concave, then $\bar{U} = \lambda \bar{U}^w + (1-\lambda)U^e$ is strictly concave. The unique size threshold a^x_{gp} that maximizes $\bar{U}$ is then given by (5.11).

Proof of Item 4

Finally, from propositions 1 and 2, $\frac{\partial U^w(a)}{\partial w} \geq \frac{\partial U^e(a)}{\partial w}$ for $a > \underline{a}_0$. Therefore, the size threshold at which $\frac{\partial \bar{U}^w}{\partial a^x} = 0$ is to the left of that at which $\frac{\partial \bar{U}^e}{\partial a^x} = 0$. Since both functions are concave, the size threshold that maximizes $\bar{U}$ moves to the left as λ increases, which proves the last item. ■

Lemma 3 *The expected labor regulation policy, $\mathcal{P}_{rp} : [\underline{a}_0, a_M] \to \mathcal{O}$ that arises from the random proposer model satisfies,*

$$\mathcal{P}^{\varphi}_{rp}(a) = \begin{cases} \varphi_0 & \text{if } a \in [\underline{a}_0, \tilde{a}^{\varphi}_0), \\ \varphi_0 + \mu\Delta & \text{if } a \geq \tilde{a}^{\varphi}_0, \end{cases}$$

and

$$\mathcal{P}^{\theta}_{rp}(a) = \begin{cases} \theta_0 & \text{if } a \in [\underline{a}_0, \tilde{a}^{\theta}_0), \\ \theta_0 + \mu\Delta & \text{if } a \geq \tilde{a}^{\theta}_0. \end{cases}$$

Proof: Define $\mathcal{P}_u(a) = (\mathcal{P}^{\varphi}_u(a), \mathcal{P}^{\theta}_u(a))$ and $\mathcal{P}_e(a) = (\mathcal{P}^{\varphi}_e(a), \mathcal{P}^{\theta}_e(a))$ as the preferred policies of unions and entrepreneurs, respectively. First, observe that when bargaining, agents cannot anticipate the effect of their decisions on the equilibrium wage, w. Thus, in this case, $w_\varphi = psw$ and $w_\theta = (1-p)w$. That is, they only consider the direct positive effect of higher labor protection on the expected wage, but not the negative effect on w that happens when the economy-wide labor regulations improve. From proposition 2, $\frac{dU^w(|\varphi,\theta)}{dx} < 0$ if $a \in [\underline{a}_0, \tilde{a}^x)$ and $\frac{dU^w(|\varphi,\theta)}{dx} > 0$ if $a > \tilde{a}^x$. Thus,

$$\mathcal{P}_u^{\varphi}(a) = \begin{cases} \varphi_0 & \text{if } a \in [\underline{a}_0, \tilde{a}_0^{\varphi}), \\ \varphi_1 & \text{if } a \geq \tilde{a}_0^{\varphi}, \end{cases}$$

and

$$\mathcal{P}_u^{\theta}(a) = \begin{cases} \theta_0 & \text{if } a \in [\underline{a}_0, \tilde{a}_0^{\theta}), \\ \theta_1 & \text{if } a \geq \tilde{a}_0^{\theta}, \end{cases}$$

Moreover, from proposition 1, $\frac{\partial U^e(a|\mathcal{P}_0)}{\partial x} < 0$ for any $a \geq \underline{a}_0$, thus $\mathcal{P}_e^{\varphi}(a) = \varphi_0$ and $\mathcal{P}_e^{\theta}(a) = \theta_0$.

From the random proposer model, the labor regulation is set at $\mathcal{P}_u(a)$ with frequency μ and at $\mathcal{P}_e(a)$ with frequency $1-\mu$. Thus, the resulting expected labor rule $\mathcal{P}_{rp} = (\mathcal{P}_{rp}^{\varphi}, \mathcal{P}_{rp}^{\theta})$ is given by

$$\mathcal{P}_{rp}^{\varphi}(a) = \begin{cases} \varphi_0 & \text{if } a \in [\underline{a}_0, \tilde{a}_{rp}^{\varphi}), \\ \varphi_1\mu + \varphi_0(1-\mu) & \text{if } a \geq \tilde{a}_{rp}^{\varphi}, \end{cases}$$

and

$$\mathcal{P}_{rp}^{\theta}(a) = \begin{cases} \theta_0 & \text{if } a \in [\underline{a}_0, \tilde{a}_{rp}^{\theta}), \\ \theta_1\mu + \theta_0(1-\mu) & \text{if } a \geq \tilde{a}_{rp}^{\theta}, \end{cases}$$

Using that $\varphi_1 = \varphi_0 + \Delta$ and $\theta_1 = \theta_0 + \Delta$ leads to expressions (6.10) and (6.11). ■

Proposition 6 *The union's bargaining power function, $\mu(\lambda)$ that implements the maximum asset-based welfare is as follows,*

$$\mu(\lambda) = \begin{cases} 0 & \text{if } \lambda \leq \frac{1}{2+1/(\gamma-2)}, \\ \chi(\lambda) & \text{if } \lambda \in (\tilde{\lambda}, 1], \end{cases}$$

where $\chi(\lambda) \in (0,1]$ is some increasing function in λ such that $\chi(1) = 1$ and $\tilde{\lambda} > \frac{1}{2-1/\gamma}$.

Proof: Define the weighted welfare of the preferred policy given λ as follows,

$$\tilde{U}(\lambda) \equiv \max_{a^x \in (\underline{a}_0, a_M)} \left\{ \lambda \cdot \left(\int_{\underline{a}_0}^{a^x} U^w(a|x_0)\partial G + \int_{a^x}^{a_M} U^w(a|x_1)\partial G \right) + (1-\lambda) \cdot \left(\int_{\underline{a}_0}^{a^x} U^e(a|x_0)\partial G + \int_{a^x}^{a_M} U^e(a|x_1)\partial G \right) \right\}. \tag{B.29}$$

Define the weighted welfare of the expected labor regulation policy ($\mathcal{P}_{rp}$) given λ and bargaining power μ as,

$$V(\lambda,\mu) = \lambda\cdot\left(\int_{\underline{a}_0}^{\tilde{a}_0^x} U^w(a|x_0)\partial G + \int_{\tilde{a}_0^x}^{a_M} U^w(a|\tilde{x}_1)\partial G\right)+(1-\lambda)\cdot\left(\int_{\underline{a}_0}^{\tilde{a}_0^x} U^e(a|x_0)\partial G + \int_{\tilde{a}_0^x}^{a_M} U^e(a|\tilde{x}_1)\partial G\right), \tag{B.30}$$

where $\tilde{x}_1 \equiv x_0+\mu\cdot\Delta$. First, note that from lemma 3, when $\lambda = 1$ and $\mu = 1$, then the size thresholds arising from the random proposer model are $(\tilde{a}^{\varphi}, \tilde{a}^{\theta})$, which coincide with the preferred policy of the government. Thus, we have that $\tilde{U}(1) = V(1,1)$. That is, $\mu = 1$ implements $\tilde{U}(1)$. Secondly, observe that if $\mu = 0$, then $\mathcal{P}_{rp} = (\varphi_0, \theta_0)$ which coincides with $\mathcal{P}_{gp}$ given $\lambda \leq \frac{1}{2+1/(\gamma-2)}$. Therefore, $\mu = 0$ implements $\tilde{U}(\lambda)$ for any $\lambda \leq \frac{1}{2+1/(\gamma-2)}$.

Finally, all is left to do is to find what μ implements $\tilde{U}(\lambda)$ when $\lambda > \frac{1}{2-1/\gamma}$. Define the FOC (B.13) as a function of (λ, μ, a),

$$FOC(\lambda,\mu,a) \equiv \lambda\frac{\partial U^w(a|\tilde{x}_1)}{\partial x} + (1-\lambda)\frac{\partial U^e(a|\tilde{x}_1)}{\partial x}. \tag{B.31}$$

Additionally, differentiate $V(\lambda,\mu)$ in terms of μ,

$$\begin{aligned}
\frac{\partial V(\lambda,\mu)}{\partial\mu} &= \frac{\partial\tilde{x}_1}{\partial\mu}\left(\lambda\int_{a^x}^{a_M}\frac{\partial U^w(a|\tilde{x}_1)}{\partial x}\partial G + (1-\lambda)\int_{a^x}^{a_M}\frac{\partial U^e(a|\tilde{x}_1)}{\partial x}\partial G\right),\\
&= \Delta\left(\int_{\tilde{a}_0^x}^{a_M}\lambda\frac{\partial\tilde{U}(a|\tilde{x}_1)}{\partial x} + (1-\lambda)\frac{\partial U^e(a|\tilde{x}_1)}{\partial x}\partial G\right), &\text{(B.32)}\\
&= \Delta\int_{\tilde{a}_0^x}^{a_M} FOC(\lambda,\mu,a)\partial G. &\text{(B.33)}
\end{aligned}$$

Pick $\lambda = 1-\varepsilon$, for some $\varepsilon > 0$, but small. Note that $FOC(1-\varepsilon, 1, a) < 0$ if $a > a^x_{gp}$. Thus, by continuity of $FOC(\lambda,\mu,a)$, there must be some $\epsilon \in (0,1)$ such that $\frac{\partial V(\lambda,\mu)}{\partial\mu} < 0$ for $\mu \in (1-\epsilon, 1)$. In consequence, it must be that $V(1-\varepsilon,\mu) \geq V(1-\varepsilon,1) = \tilde{U}(1-\varepsilon)$ for some $\mu \in (1-\epsilon,1)$. Hence, for a given $\lambda = 1-\varepsilon$, there exists some $\mu(\lambda) \in (1-\epsilon,1)$ that implements $\tilde{U}(1-\varepsilon)$. Since $\tilde{U}(\lambda)$ is increasing in λ, it must be that the function characterizing $\mu(\lambda)$, named as $\chi(\lambda)$, is increasing in λ. Finally, since ε must be small, this result applies to some neighbourhood $\lambda \in (\tilde{\lambda}, 1)$, where $\tilde{\lambda} > \frac{1}{2-1/\gamma}$. This concludes the proof. ■

C Appendix: Data

This section explains how the data presented in figures 1 and 2 was constructed. I list below the sources for each of the 25 countries. Labor codes were obtained mainly from the International Labor Organization (ILO). For some countries the information comes from studies regarding labor regulations (which are cited after those countries' names). The focus is on countries that apply S-shaped EPLs. Thus, the data is on the size threshold from which EPLs become stricter. For each country, I searched the year in which the size threshold was enacted and all the instances in which it was changed. I consider both individual and collective dismissal regulations.

Left and right-wing governments are defined on the basis of the political orientation of the executive as measured by the World Bank Database of Political Institutions (WDPI), and defined in Beck et al. (2001). The WDPI provides a variable that can take three values "Left", "Center" or "Right". There are only two instances in which a size threshold was enacted by a center government: in 1960, Italy and in 2007, Finland.

Argentina According to the Small and Medium Entreprises Law (SMEL) enacted in 1995, article 83, the rules on notice period don't apply to SMEs defined as those companies with less than 40 employees.

Australia According to the Workplace Relations Act, 2005, claims of unfair dismissal were not available for workers in firms with 100 or more workers. Four years later, the Fair Work Act 2009, defined exemptions pertaining to dismissal in firms with less than 15 employees. Source: Vranken (2005).

Austria The Work Constitution Act, 1973, establishes that protection regarding individual dismissal only applies to firms with more than 5 employees. According to section 45a of the Labour Market Promotion Act, 1969, the definition of collective dismissals excluded enterprises with less than 20 workers. Since there are size thresholds from which both individual and collective dismissal regulations apply I choose to use the one reported by ILO, i.e. 5.

Belgium According to article 1, Royal Order on Collective Dismissals, 1976, collective dismissal regulations apply to firms with more than 20 workers. However, individual dismissal regulations apply to all firms.

Bulgary According to the Labor Code, 1986, enterprises with less than 20 workers are excluded from collective dismissal regulations. Individual dismissal regulations apply to all firms.

Cyprus The Collective Dismissals Act, section 2, 2001, excludes firms with less than 20 employees from collective dismissal regulations. Individual dismissal regulations apply to all firms.

Czech Republic According to section 62 of the Labor Code, 2006, enterprises with less than 20 workers are excluded from collective dismissal regulations. Individual dismissal regulations apply to all firms.

Denmark According to section 1 of the Collective Dismissals Act, 1994, enterprises with less than 20 workers are excluded from collective dismissal regulations. Individual dismissal regulations apply to all firms.

France Labor laws make special provisions for firms with more than 10, 11, 20 or 50 employees. However, 50 is generally agreed by labor lawyers to be the threshold from which costs increase significantly. According to the Labor Code, articles L.1235-10 to L.1235-12, 1973, firms with at least 50 employees firing more than 9 workers must follow a complex redundancy plan with oversight from Ministry of Labor. Sources: Garicano et al. (2016), Gourio and Roys (2014).

Finland The Act on Cooperation within Undertakings, 2007, establishes that procedures with regards to economic dismissals apply only to firms with 20 or more workers.

Germany In 1951, the Federal Parliament enacted a federal Act on the Protection against Dismissal (Kündigungsschutzgesetz, PADA). The Act provided that dismissals in establishments with more than 5 workers required a social justification. The threshold for the applicability of the PADA has changed three times. In 1996, from 5 to 10 employees and then back again to 5 workers in 1999. Since 2004 this threshold has been shifted to 10 workers. Sources: Siefert and Funken-Hotzel (2003), Verick (2004), Bellmann et al. (2014).

Greece According to Act No. 1387/1983 enterprises with less than 20 workers are excluded from collective dismissal regulations. Individual dismissal regulations apply to all firms.

Hungary According to section 94 of the Labor Code, 1992, enterprises with less than 20 workers are excluded from collective dismissal regulations. Individual dismissal regulations apply to all firms.

Italy Individual dismissals were first regulated in Italy in 1966 through Law No. 604. In case of dismissal, workers could take employers to court. If judges ruled that these dismissals were unfair, employers had either to reinstate the worker or pay a firing cost which depended on firm size. Those firms with more than 60 employees had to pay twice the amount paid by those firms with less than 60 workers. In 1970, the Workers' Statute (Law No. 300) established that in case of unfair dismissal those firms with more than 15 employees had to reinstate workers and pay their foregone wages. Sources: Kugler and Pica (2008), Rutherford and Frangi (2018)

South Korea The Labour Standards Act enacted in 1997, article 11, establishes that employment regulations apply to firms with more than 5 workers. Source: Yoo and Kang (2012).

Kyrgyzstan According to article 55 of the Labor Code, 2004, fixed-term contracts may be concluded during the first year of its creation in enterprises employing up to 15 workers.

Montenegro According to article 92 of the Labor Law, 2008, regulations on collective dismissals apply only to firms with at least 20 employees.

Morroco According to article 66 of the Labor Code, 2003, regulations on collective dismissals apply only to firms with at least 10 employees. Individual dismissal regulations apply to all firms.

Portugal The Decreto-Lei 64-A/89 introduced in 1989 softened the dismissal constraints faced by firms. Article 10 defined 12 specific rules that firms with more than 20 workers needed to follow. Only four of these rules applied to firms employing 20 or fewer workers. Firms with less than 50 employees were allowed to conduct a collective dismissal involving only two workers, but those enterprises with more than 50 workers required that at least five workers be dismissed. Source: Martins (2009).

Romania Article 1 of the Labor Code, 2004, that regulated individual and collective dismissal excluded enterprises with less than 20 employees.

Slovakia A new definition of collective dismissals was introduced in 2011 into the Labor Code. According to section 73, enterprises with less than 20 workers are excluded from procedural requirements regarding collective dismissals.

Slovenia The Employment Relationship Act (ERA), 2002, excluded firms with less than 20 employees from the procedural requirements applicable to collective dismissals.

Turkey According to article 18 of the Labor Act, 2003, workers in establishments with less than 30 employees are not covered by the job security provision.

United States According to the Workforce Investment Act passed in 1989, firms with 100 or more employees, excluding part-time employees, are required to provide 60 days' written notice to displaced workers. Source: Addison and Blackburn (1994).

Venezuela Under the Organic Labor Law of 1990, enterprises with less than 10 employees were exempt from the obligation to reinstate workers even if there was a court decision ruling that the dismissal was unjustified.

D Appendix: Additional Proofs and Extensions

D.1 Political mechanism

This section presents a politico-economy microfoundation for the political equilibrium described in the paper. I show that the government's problem (presented in section 3.4) can be rationalized as a probabilistic voting model along the lines of Persson and Tabellini (2000, pp. 52-58), where the political weight λ depends on the primitives of the model. Figure 20 illustrates the time line.

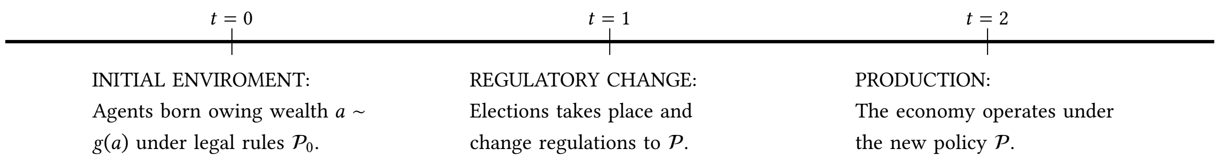

Figure 20: Timeline.

As shown in section 3.3, given $\mathcal{P}_0$, there are two groups of voters: workers (W) with wealth $a < \underline{a}_0$, and entrepreneurs (E) with $a \geq \underline{a}_0$. Their utilities are represented by (3.2) and (3.4), respectively. The political preferences of agents are defined on the basis of the ex-ante competitive equilibrium. That is, given $\mathcal{P}_0$ and a, agents vote understanding what their position in society would be and how an improvement of EPLs would affect them relative to this initial position.

The electoral competition takes place between two parties, A and B. Both parties simultaneously and noncooperatively announce their electoral platforms, $\mathcal{P}_A$ and $\mathcal{P}_B$, subject to the labor market equilibrium condition (3.14). The policies $\mathcal{P}_A$ and $\mathcal{P}_B$ map firm's assets to a specific strength of EPLs (x_0 or x_1, with $x \in \{\varphi, \theta\}$). Thus the proposed political platform of the parties is constrained to the set of functions: $\mathcal{P} : [0, a_M] \to \Theta$, where $\Theta \equiv \{(\varphi_0, \theta_0), (\varphi_1, \theta_0), (\varphi_0, \theta_1), (\varphi_1, \theta_1)\}$ is the set of EPLs that can be implemented at each firm.

Under a multidimensional policy, Downsian electoral competition is known to produce cycling problems that arise because parties' objective functions are discontinuous in the policy space. Probabilistic voting smooths the political objective function by introducing uncertainty from the parties point of view (Lindbeck and Weibull, 1987). Specifically, there is uncertainty about the political preferences of each voter. As in Fischer and Huerta (2021), I assume there is a continuum of agents (a, ν). Voter (a, ν) in group $j \in \{W, E\}$ votes for party A if:

$$U^j(a|\mathcal{P}_A) > U^j(a|\mathcal{P}_B) + \delta + \sigma^j_\nu(a), \tag{D.1}$$

where δ reflects the general popularity of party B, which is assumed to be uniformly distributed on $[-1/(2\psi), 1/(2\psi)]$. The value of δ becomes known after the policy platforms have been announced. Thus parties announce their policy platforms under uncertainty about the results of the election. The variable $\sigma_v^j(a)$ represents the ideological preference of voter (a, v) for party B. The distribution of $\sigma_v^j(a)$ differs across workers and entrepreneurs, which is assumed to be uniform on $[-1/(2\chi^j), 1/(2\chi^j)]$. Note that neither group is biased towards either party, but that they differ in their ideological homogeneity represented by the density χ^j. Parties know the group-specific ideological distributions before announcing their platforms. The term $\delta + \sigma_v^j(a)$ captures the relative 'appeal' of candidate B. That is, the inherent bias of voter v with wealth a in group j for party B, irrespective of the proposed political platforms.

I study the policy outcome under an electoral rule corresponding to proportional representation. Thus a party requires more than 50% of total votes to win the election. To characterize the political outcome, it is useful to identify the 'swing voter' ($v = V$) in each group $j \in \{W, E\}$ and for each value of wealth a in that group. That is, the voter in group j with wealth a who is indifferent between the two parties:

$$\sigma_V^j(a) = U^j(a|\mathcal{P}_A) - U^j(a|\mathcal{P}_B) - \delta. \tag{D.2}$$

All agents endowed with wealth a whose ideological preference is such that $\sigma_v^j(a) < \sigma_V^j(a)$ vote for party A, while the rest vote for party B. Therefore, conditional on δ, the fraction of agents in group j with wealth a that vote for party A is:

$$\begin{aligned} \pi_A^j(a|\delta) &= Prob[\sigma_v^j(a) < \sigma_V^j(a)], \\ &= \chi^j[U^j(a|\mathcal{P}_A) - U^j(a|\mathcal{P}_B) - \delta] + \frac{1}{2}. \end{aligned} \tag{D.3}$$

The probability that party A wins the election, p_A is then given by

$$p_A = Prob\left[\int_0^{\underline{a}_0} \pi_A^W(a|\delta)\partial G(a) + \int_{\underline{a}_0}^{a_M} \pi_A^E(a|\delta)\partial G(a) \geq \frac{1}{2}\right],$$

where the probability is taken with respect to the general popularity measure δ. Rearranging terms leads to:

$$\begin{aligned} p_A = Prob\Bigg[&\chi^W \int_0^{\underline{a}_0} [U^W(a|\mathcal{P}_A) - U^W(a|\mathcal{P}_B)]\partial G(a) + \chi^E \int_{\underline{a}_0}^{a_M} [U^E(a|\mathcal{P}_A) - U^E(a|\mathcal{P}_B)]\partial G(a) \\ &- \delta[\chi^W G(\underline{a}_0) + \chi^E(1 - G(\underline{a}_0))] \geq 0\Bigg], \\ = Prob&\left[\delta \leq \frac{\chi^W \int_0^{\underline{a}_0}[U^W(a|\mathcal{P}_A) - U^W(a|\mathcal{P}_B)]\partial G(a) + \chi^E \int_{\underline{a}_0}^{a_M}[U^E(a|\mathcal{P}_A) - U^E(a|\mathcal{P}_B)]\partial G(a)}{\chi^W G(\underline{a}_0) + \chi^E(1 - G(\underline{a}_0))}\right], \\ = Prob&\left[\delta \leq \frac{\chi^W[\bar{U}^W(\mathcal{P}_A) - \bar{U}^W(\mathcal{P}_B)] + \chi^E[\bar{U}^E(\mathcal{P}_A) - \bar{U}^E(\mathcal{P}_B)]}{\bar{\chi}}\right], \end{aligned}$$

where I have defined:

$$\bar{U}^W(\mathcal{P}) \equiv \int_0^{\underline{a}_0} U^W(a|\mathcal{P})\partial G(a),$$
$$\bar{U}^E(\mathcal{P}) \equiv \int_{\underline{a}_0}^{a_M} U^E(a|\mathcal{P})\partial G(a),$$
$$\bar{\chi} \equiv \chi^W G(\underline{a}_0) + \chi^E(1 - G(\underline{a}_0)).$$

Therefore, the probability that party A wins the election is,

$$p_A = \psi\left[\frac{\chi^W}{\bar{\chi}}(\bar{U}^W(\mathcal{P}_A) - \bar{U}^W(\mathcal{P}_B)) + \frac{\chi^E}{\bar{\chi}}(\bar{U}^E(\mathcal{P}_A) - \bar{U}^E(\mathcal{P}_B))\right] + \frac{1}{2}$$

Define the relative political weight of workers and entrepreneurs by $\lambda^W \equiv \psi\frac{\chi^W}{\bar{\chi}}$ and $\lambda^E \equiv \psi\frac{\chi^E}{\bar{\chi}}$, respectively. Since both parties maximize the probability of wining the election, the Nash equilibrium is characterized by

$$\mathcal{P}_A^* = \arg\max_{\mathcal{P}_A}\{\lambda^W(\bar{U}^W(\mathcal{P}_A) - \bar{U}^W(\mathcal{P}_B)) + \lambda^E(\bar{U}^E(\mathcal{P}_A) - \bar{U}^E(\mathcal{P}_B))\}$$
$$\mathcal{P}_B^* = \arg\max_{\mathcal{P}_B}\{\lambda^W(\bar{U}^W(\mathcal{P}_B) - \bar{U}^W(\mathcal{P}_A)) + \lambda^E(\bar{U}^E(\mathcal{P}_B) - \bar{U}^E(\mathcal{P}_A))\}$$

As a result, the two parties' platforms converge in equilibrium to the same policy function $\mathcal{P}^*$ that maximizes the weighted welfare of workers and entrepreneurs,

$$\mathcal{P}^* = \arg\max_{\mathcal{P}}\{\lambda^W\bar{U}^W(\mathcal{P}) + \lambda^E\bar{U}^E(\mathcal{P})\}, \tag{D.4}$$

subject to the labor market equilibrium condition (3.14).

In order to interpret problem (D.4), rewrite the political weights as follows,

$$\lambda^W = \frac{\psi}{G(\underline{a}_0) + \frac{\chi^E}{\chi^W}(1 - G(\underline{a}_0))},$$
$$\lambda^E = \frac{\psi}{\left(\frac{\chi^W}{\chi^E} - 1\right)G(\underline{a}_0) + 1}.$$

Note that the political weights depend on both exogenous and endogenous variables. First, they are a function of the dispersion of the ideological preferences of both groups, measured by χ^j. Secondly, they are a function of the variability of party's B general popularity, ψ. Finally, they depend on the minimum wealth to obtain a loan, $\underline{a}_0$ under the initial policy $\mathcal{P}_0$. As explained in section 3.3, that threshold is endogenously determined as a function of the primitives of the model. [38]

The political weights λ^j have an structural interpretation: they measure the relative dispersion of ideological preferences within group j. The ratio χ^W/χ^E determines the number of swing voters in each group. For instance, when χ^W increases then the political weight of workers λ^W increases, but λ^E decreases. Intuitively, workers become more responsive to EPLs announcements in favor or against them. As a result, the vote of entrepreneurs become less responsive to EPLs announcements compared to workers. Thus workers become more politically powerful relative to entrepreneurs and the equilibrium platform becomes more pro-worker.

In order to write problem (D.4) as in section 3.4, I normalize the political weights by choosing $\psi = \frac{\chi^w G(\underline{a}_0)+\chi^E(1-G(\underline{a}_0))}{\chi^W+\chi^E}$. Thus, $\lambda^W + \lambda^E = 1$. Define $\lambda \equiv \lambda^E$, then the problem can be rewritten as

$$\mathcal{P}^* = \arg\max_{\mathcal{P}}\{\lambda \bar{U}^W(\mathcal{P}) + (1-\lambda)\bar{U}^E(\mathcal{P})\},$$

subject to the labor market equilibrium condition (3.14).

This corresponds to the 'politician's problem' presented in the body of the paper. Thus when λ increases, the representative government chooses a policy platform that favors relatively more workers (pro-worker). If λ decreases the government becomes more pro-entrepreneurs. In particular, when $\chi^W \to +\infty$ then $\lambda \to 1$ and the government weights only workers. In contrast, if $\chi^E \to +\infty$ then $\lambda \to 0$ and the government cares only about entrepreneurs.

D.2 Two-dimensional labor reform

This section deals with a two-dimensional labor reform. That is, the government can change both individual and collective dismissal regulations. From proposition 3, problem (3.14) reduces to finding two size thresholds, a^φ and a^θ, from which EPLs become stricter. To simplify exposition define, $a^1 \equiv \min\{a^\varphi, a^\theta\}$ and $a^2 \equiv \max\{a^\varphi, a^\theta\}$. Further, define,

$$(\tilde{\varphi}, \tilde{\theta}) \equiv (\varphi_1, \theta_0)\mathbf{1}[a^\varphi \geq a^\theta] + (\varphi_0, \theta_1)\mathbf{1}[a^\varphi < a^\theta].$$

Thus, aggregate entrepreneurs' welfare ($\lambda = 0$) is,

$$\bar{U}^e(a^\varphi, a^\theta) = \int_{\underline{a}_0}^{a^1} U^e(a|\varphi_0, \theta_0)\partial G + \int_{a^1}^{a^2} U^e(a|\tilde{\varphi}, \tilde{\theta})\partial G + \int_{a^2}^{a_M} U^e(a|\varphi_1, \theta_1)\partial G,$$

[38] Specifically, $\underline{a}_0$ depends on: i) the probability of success of a firm p, ii) the recovery rate of bankruptcy procedures η, iii) the initial strength of EPLs (φ_0, θ_0), iv) the international interest rate ρ, v) the fixed cost F to start a firm, and vi) the parameters of the production function α, β.

while aggregate workers' welfare ($\lambda = 1$) is given by

$$\bar{U}^w(a^\varphi, a^\theta) = \int_{\underline{a}_0}^{a^1} U^w(a|\varphi_0, \theta_0)\partial G + \int_{a^1}^{a^2} U^w(a|\tilde{\varphi}, \tilde{\theta})\partial G + \int_{a^2}^{a_M} U^w(a|\varphi_1, \theta_1)\partial G.$$

The government's problem is written as follows,

$$\max_{(a^\varphi, a^\theta) \in [\underline{a}_0, a_M]^2} \{\bar{U}(a_1, a_2) \equiv \lambda \bar{U}^w(a_1, a_2) + (1-\lambda)\bar{U}^w(a_1, a_2)\}$$

$$\begin{aligned} s.t \qquad m(\varphi_0, \theta_0) \cdot l_s(\varphi_0, \theta_0) &= \int_{\underline{a}_0}^{a^1} l(a|\varphi_0, \theta_0)\partial G, \\ m(\tilde{\varphi}, \tilde{\theta}) \cdot l_s(\tilde{\varphi}, \tilde{\theta}) &= \int_{a^1}^{a^2} l(a|\tilde{\varphi}, \tilde{\theta})\partial G, \\ m(\varphi_1, \theta_1) \cdot l_s(\varphi_1, \theta_1) &= \int_{\underline{a}_0}^{a^1} l(a|\varphi_1, \theta_1)\partial G, \\ \sum_{(\varphi, \theta) \in \Theta} m(\varphi, \theta) &= G(\underline{a}_0), \end{aligned}$$

where $m(\varphi, \theta)$ corresponds to the mass of workers subject to EPLs $(\varphi, \theta) \in \Theta$ and recall that a^1 and a^2 are defined in terms of (a^φ, a^θ). The first three conditions equalize labor supplied and demanded under the different EPL regimes. The final condition asks that the sum of workers under different EPLs must equal the total mass of workers, $G(\underline{a}_0)$. As in the unidimensional case, these conditions uniquely define $m(\varphi, \theta) \in \Theta$ and the equilibrium wage w. The following proposition describes the equilibrium policy under flexible wages.

Proposition 7 *$\bar{U}(a^\varphi, a^\theta, \lambda)$ achieves a global maximum in $[\underline{a}_0, a_M]^2$ at some size thresholds $a^\varphi_{gp} \in (\underline{a}_0, a_M)$ and $a^\theta_{gp} \in (\underline{a}_0, a_M)$ characterized by*

$$(a^\varphi_{gp}, a^\theta_{gp}) = \sup_{(a^\varphi, a^\theta)} \bar{U}(a^\varphi, a^\theta, \lambda). \tag{D.5}$$

Proof: The same arguments used to prove item 1 of proposition 5 apply in the two-dimensional case. Thus, $\bar{U}(a^\varphi, a^\theta)$ is a bounded and continuous function in $[\underline{a}_0, a_M]^2$, satisfying:[39]
i) $\bar{U}(\underline{a}_0, \underline{a}_0) = \bar{U}(a_M, a_M) > 0$, ii) $\frac{\partial \bar{U}(\underline{a}_0^+, a^\theta)}{\partial a^\varphi} > 0, \forall a^\theta \in [\underline{a}_0, a_M]$ and iii) $\frac{\partial \bar{U}(a^\varphi, \underline{a}_0^+)}{\partial a^\theta} > 0, \forall a^\varphi \in [\underline{a}_0, a_M]$.

In consequence, $\bar{U}(a^\varphi, a^\theta)$ achieves a global maximum. Moreover, properties i) to iii) imply that the global maximum is achieved at some $a^\varphi_{gp} \in (\underline{a}_0, a_M)$ and $a^\theta_{gp} \in (\underline{a}_0, a_M)$. ■

As in the unidimensional case, the proposition states that the equilibrium policy is S-shaped in both dimensions regardless of the political orientation of the government. Thus, in equilibrium there are three possible regulatory regimes: (φ_0, θ_0), $(\tilde{\varphi}, \tilde{\theta})$ and (φ_1, θ_1).

[39] I omit the dependece of $\bar{U}$ on λ to simplify notation.

Figure 21 illustrates the case in which $a^{\varphi}_{gp} > a^{\theta}_{gp}$, i.e. $(\tilde{\varphi}, \tilde{\theta}) = (\varphi_1, \theta_0)$. First, smaller firms with assets $a \in [\underline{a}_0, a^{\varphi}_{gp})$ are subject to both low individual and low collective dismissal regulations, (φ_0, θ_0). There is a range of medium-sized firms with assets $a \in [a^{\varphi}_{gp}, a^{\theta}_{gp})$ that face stronger individual regulations, but weak collective dismissal regulations, (φ_1, θ_0). Finally, larger firms with $a > a^{\theta}_{gp}$ are subject to stronger individual and collective EPLs, (φ_1, θ_1).

This EPLs design illustrates the cases of Austria and France. In the case of Austria, individual dismissal regulations apply only to firms with more than 5 employees, while collective regulations exclude firms with less than 20 workers. In France, firms with more than 10 workers are subject to stricter EPLs regarding economic dismissal. Additionally, in case of firing more than 9 workers (collective dismissal) firms with more than 50 workers must follow a special legal process which increases dismissal costs.

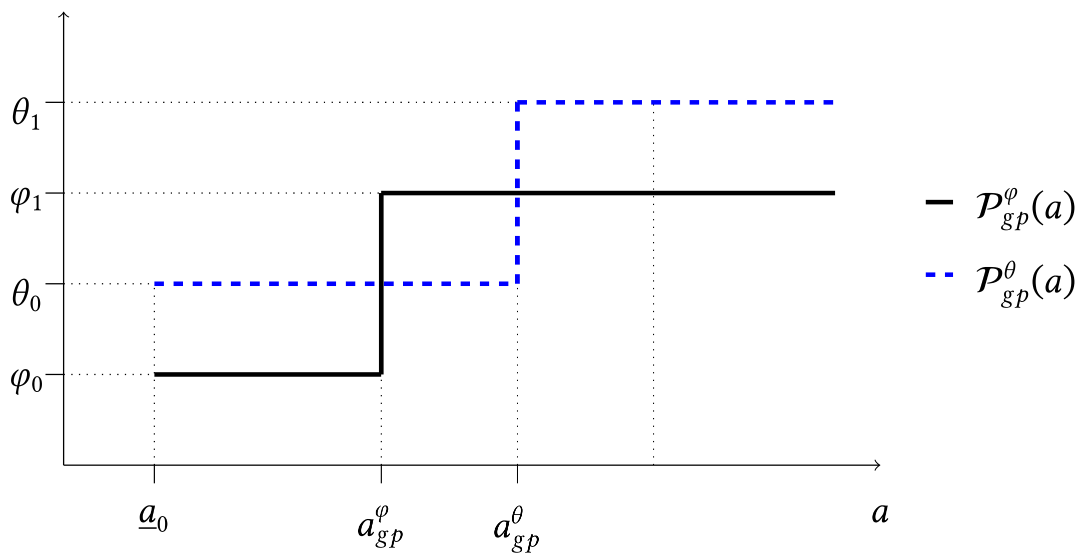

Figure 21: Equilibrium labor policy, $\mathcal{P}_{gp}(a) = (\mathcal{P}^{\varphi}_{gp}(a), \mathcal{P}^{\theta}_{gp}(a))$.

D.3 Political affiliations

As shown in section 5.2, depending on the political orientation of the government, different labor regulation policies are selected. Therefore, whether the policy-maker is left or right-wing matters in terms of ex-post welfare for each group of agents. In this section, I study the political affiliations of the different groups of agents if they can anticipate the policy to be implemented by a leftist $(\lambda = 1)$ or right-wing $(\lambda = 0)$ government. I focus on the case with flexible wages which is more interesting. Given the initial EPL, $\mathcal{P}_0$ agents can anticipate the equilibrium policy that a left or right-wing government will implement at $t = 1$, and thus, their ex-post expected welfare at $t = 2$.

The political affiliations of the different interest groups as function of their firms assets are summarized in figure 22. There are three cases depending on the location of $\tilde{a}_0^x$, as illustrated by panels a) to c). In the figure, 'W' and 'E' stand for 'workers' and 'entrepreneurs', respectively. 'LW' and 'RW' stand for 'left-wing' and 'right-wing', respectively.

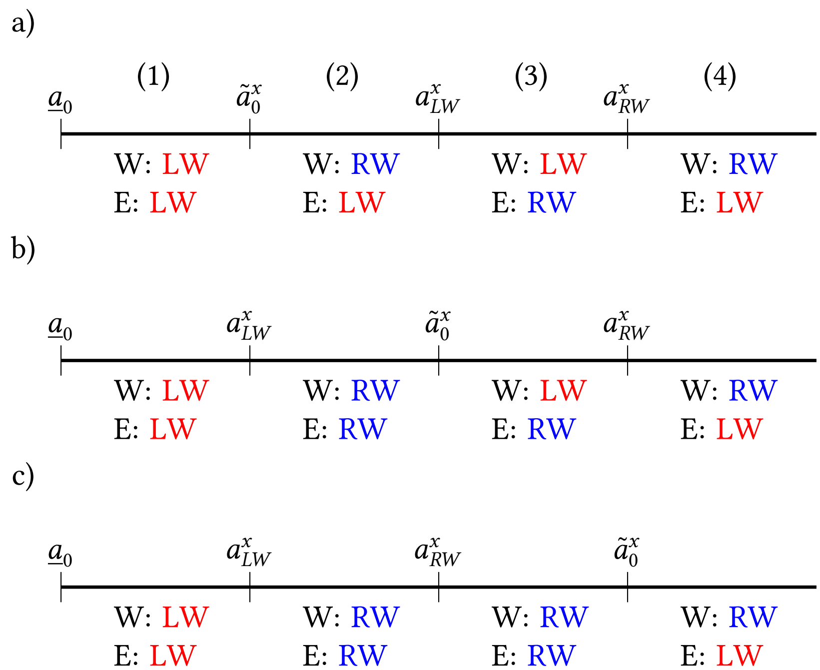

Figure 22: Political affiliations.

First, the figure shows that there are four ranges of agents with different political affiliations, enumerated as 1, 2, 3 and 4. In any case there are two groups of workers that have opposing interests. Those matched to the smallest firms (group 1) support a left-wing labor policy as opposed to those in largest firms (group 4). The intuition is as follows. Workers in group 1 don't want protection, since a higher expected wage hurts their firms which are forced to shrink and hire less labor. A left-wing government provides protection to a large set of workers, but not to those in the smallest firms (those in group 1). This pushes down the equilibrium wage benefiting the smallest firms and thus, their workers. Workers in group 4 can anticipate that even the most right-wing government will protect them. Thus, they are against more leftist governments that set a lower size threshold which leads to a lower wage and hurts them.

Secondly, there is a middle class of workers and entrepreneurs with heterogeneous political preferences (groups 2 and 3). In panel a), when $\tilde{a}_0^x < a_{LW}^x$, workers in firms with $a \in [\tilde{a}_0^x, a_{LW}^x)$ know that even the most leftist government is not going to provide them with higher protection. Thus, since they are better off under a higher expected wage, they support a right-wing government which sets a lower size threshold. As opposed to their interests, entrepreneurs running those firms support a leftist government which is not going to impose higher EPLs to their firms, but is going to do so for the rest of the firms, leading to a lower equilibrium wage. On the other hand, the political preferences are reversed for agents in firms with $a \in (a^x_{LW}, a^x_{RW})$. In this case workers can receive higher protection if they support a left-wing government, but their entrepreneurs suffer from higher wages. Interestingly, as $\tilde{a}^x_0$ increases relative to a^x_{LW} and a^x_{RW} (panel b) and panel c)), fewer workers want protection and more middle-class agents support a right-wing government.

Overall, the model predicts heterogeneous political preferences across groups of workers and entrepreneurs. Those agents in the smallest and largest firms have well-defined political affiliations. However, there is middle-class with heterogeneous preferences depending on the different configurations of the parameters. Cross class coalitions arise in equilibrium.

D.4 Labor-based policy

This section shows that the equilibrium EPL remains S-shaped under a labor-based policy. I start by showing that the equilibrium policy satisfies monotonicity at each component.

Proposition 8 *Any labor regulation policy, $\mathcal{P}$ that solves* (6.1) *satisfies monotonicity at each component,*

$$\mathcal{P}^x(l) \,:\, \mathcal{P}^x(l') \leq \mathcal{P}^x(l'') \quad \forall l' < l'', x \in \{\varphi, \theta\}.$$

Moreover, there are labor two thresholds $l^\varphi \in [0, l^\varphi_{max}]$ and $l^\theta \in [0, l^\theta_{max}]$ such that:

$$\mathcal{P}^x(l) = \begin{cases} x_0 \; if \, l < l^x, \\ x_1 \; if \, l \geq l^x. \end{cases}$$

Proof: The proof proceeds similarly to that of proposition 3. By contradiction, suppose that there is some solution to the politician's problem $\mathcal{P}^x(l)$ that violates monotonicity in some non-zero measure set $\mathcal{A} \in \mathcal{B}([l^x_{min}, l^x_{max}])$ and for which monotonicity holds in $[l^x_{min}, l^x_{max}] - \mathcal{A}$. Then, as in the proof of proposition 3, construct some alternative $\mathcal{P}^{x\prime}$ that satisfies monotonicity in $\mathcal{A}$. Denote by l^x the labor threshold above which $\mathcal{P}^{x\prime} = x_1$. Given $\mathcal{P}^{x\prime}$, there is range of firms $[a_1, a_2]$ that hire an amount of labor slightly lower than l^x:

$$U^e(a_1, d(a_1), l^x | x_0) = U^e(a_1, d(a_1), l(a_1) | x_0),$$
$$U^e(a_2, d(a_2), l^x | x_0) = U^e(a_2, d(a_2), l(a_2) | x_1).$$

Then, the labor function given assets, $\tilde{l}(a)$ for $a \in \mathcal{A}$ is,

$$\tilde{l}(a) = \begin{cases} l(a) & \text{if } a < a_1, \\ l^x & \text{if } a \in [a_1, a_2], \\ l(a) & \text{if } a > a_2. \end{cases} \tag{D.6}$$

The next step is to show that $\mathcal{P}^{x\prime}$ gives higher welfare than $\mathcal{P}^x$. This requires that $\frac{\partial}{\partial a}\left(\frac{\partial U^e}{\partial x}\right) \geq 0$ and $\frac{\partial}{\partial a}\left(\frac{\partial U^w}{\partial x}\right) \geq 0$. Note that $\frac{\partial}{\partial a}\left(\frac{\partial U^j}{\partial x}\right) = \frac{\partial}{\partial l}\left(\frac{\partial U^j}{\partial x}\right) \cdot \frac{\partial \tilde{l}(a)}{\partial a}$, where $j \in \{e, w\}$. From the proofs of propositions 1 and 2, $\frac{\partial}{\partial l}\left(\frac{\partial U^j}{\partial x}\right) > 0$. Also, $\frac{\partial l(a)}{\partial a} > 0$. Thus, from equation (D.6), $\frac{\partial \tilde{l}(a)}{\partial a} \geq 0$. Then, $\frac{\partial}{\partial a}\left(\frac{\partial U^j}{\partial x}\right) \geq 0$, which concludes the proof. ■

The next step is to map the politician's problem into a problem in which she chooses an asset threshold to maximize the labor-based welfare. Use conditions (6.3) and (6.4) to express l^x and a_2^x in terms of the asset threshold a_1^x. Formally, given a_1^x the labor threshold is $l^x = l(a_1^x|x_0)$. The second threshold $a_2^x \equiv a_2(a_1^x)$ is implicitly defined by

$$U^e(a_2^x, d(a_2^x), l(a_1^x)|x_0) = U^e(a_2^x, d(a_2^x), l(a_2^x)|x_1).$$

Then, the problem of the politician presented in section 6.1.3 can be rewritten in terms of the asset threshold a_1^x,

$$\begin{aligned}
\max_{a_1^x \in [\underline{a}_0, a_M]} \tilde{U}(a_1^x, \lambda) = \lambda \cdot \Bigg(& \int_{\underline{a}_0}^{a_1^x} U^w(a, l(a)|x_0)\partial G(a) + \int_{a_1^x}^{a_2(a_1^x)} U^w(a, l(a_1^x)|x_0)\partial G(a) + \int_{a_2(a_1^x)}^{a_M} U^w(a, l(a)|x_1)\partial G(a) \Bigg) \\
+ (1-\lambda) \cdot \Bigg(& \int_{\underline{a}_0}^{a_1^x} U^e(a, l(a)|x_0)\partial G(a) + \int_{a_1^x}^{a_2(a_1^x)} U^e(a, l(a_1^x)|x_0)\partial G(a) + \int_{a_2(a_1^x)}^{a_M} U^e(a, l(a)|x_1)\partial G(a) \Bigg)
\end{aligned}$$

$$s.t \quad m^0 \cdot l_s(x_0) = \int_{\underline{a}_0}^{a_1^x} l(a|x_0)\partial G(a) + l(a_1^x) \cdot [G(a_2(a_1^x)) - G(a_1^x)], \tag{D.7}$$

$$m^1 \cdot l_s(x_1) = \int_{a_2(a_1^x)}^{a_M} l(a|x_1)\partial G. \tag{D.8}$$

$$m^0 + m^1 = G(\underline{a}_0), \tag{D.9}$$

This alternative formulation leads to proposition 9. The proposition requires the following lemma.

Lemma 4 *The equilibrium wage w is increasing in the labor threshold l^x. In particular, if $l^x = l^x_{min}$, the change in w is such that $\frac{\partial \bar{w}}{\partial l^x} = 0$.*

Proof:

Differentiation of conditions (D.7) to (D.9) in terms of a_1^x leads to,

$$\frac{\partial m^0}{\partial a_1^x} l_s^0 + m^0 \frac{\partial l_s^0}{\partial a_1^x} = \int_{\underline{a}}^{a_1^x} \frac{\partial l^0(a)}{\partial a_1^x} \partial G + \frac{\partial l^x}{\partial a_1^x} G(a_2^x) + l^x g(a_2^x) \frac{\partial a_2^x}{\partial a_1^x} - l^0(\underline{a}) g(\underline{a}) \frac{\partial \underline{a}}{\partial a_1^x}, \tag{D.10}$$

$$\frac{\partial m^1}{\partial a_1^x} l_s^1 + m_1 \frac{\partial l_s^1}{\partial a_1^x} = \int_{a_2^x}^{a_M} \frac{\partial l^1(a)}{\partial a_1^x} \partial G - l^x g(a_2^x) \frac{\partial a_2^x}{\partial a_1^x}, \tag{D.11}$$

$$\frac{\partial m^1}{\partial a_1^x} = g(\underline{a}) \frac{\partial \underline{a}}{\partial a_1^x} - \frac{\partial m^0}{\partial a_1^x}, \tag{D.12}$$

where I have defined $l^0(a) \equiv l(a|x_0), l^1(a) \equiv l(a|x_1), l_s^0 \equiv l_s(x_0)$, and $l_s^1 \equiv l_s(x_1)$.

Combining (D.11) and (D.12),

$$\frac{\partial m^0}{\partial a^x} = \left(- \int_{a^x}^{a_M} \frac{\partial l^1(a)}{\partial a_1^x} \partial G + l^x g(a_2^x) \frac{\partial a_2^x}{\partial a_1^x} + l_s^1 g(\underline{a}) \frac{\partial \underline{a}}{\partial a^x} + m_1 \frac{\partial l_s^1}{\partial a^x} \right) \frac{1}{l_s^1}, \tag{D.13}$$

rearranging (D.10),

$$\frac{\partial m^0}{\partial a^x} = \left(\int_{\underline{a}}^{a^x} \frac{\partial l^0(a)}{\partial a^x} \partial G + \frac{\partial l^x}{\partial a_1^x} G(a_2^x) + l^x g(a_2^x) \frac{\partial a_2^x}{\partial a_1^x} - l^0(\underline{a}) g(\underline{a}) \frac{\partial \underline{a}}{\partial a_1^x} - m^0 \frac{\partial l_s^0}{\partial a_1^x} \right) \frac{1}{l_s^0}. \tag{D.14}$$

Equalizing conditions (D.13) and (D.14),

$$l_s^1 \int_{\underline{a}}^{a_1^x} \frac{\partial l^0(a)}{\partial a_1^x} \partial G + l_s^0 \int_{a_2^x}^{a_M} \frac{\partial l^1(a)}{\partial a_1^x} \partial G - l_s^1 (l^0(\underline{a}) + l_s^0) g(\underline{a}) \frac{\partial \underline{a}}{\partial a_1^x} - m^0 l_s^1 \frac{\partial l_s^0}{\partial a_1^x} - m^1 l_s^0 \frac{\partial l_s^1}{\partial a_1^x} + \frac{\partial l^x}{\partial a_1^x} G(a_2^x) = l^x (l_s^0 - l_s^1) g(a_1^x),$$

$$\Rightarrow \frac{\partial w}{\partial a_1^x} \left(l_s^1 \int_{\underline{a}}^{a_1^x} \underbrace{\frac{\partial l^0(a)}{\partial w}}_{<0} \partial G + l_s^0 \int_{a_2^x}^{a_M} \underbrace{\frac{\partial l^1(a)}{\partial w}}_{<0} \partial G - l_s^1 (l^0(\underline{a}) + l_s^0) g(\underline{a}) \underbrace{\frac{\partial \underline{a}}{\partial w}}_{<0} - m^0 l_s^1 \frac{\partial l_s^0}{\partial w} - m^1 l_s^0 \underbrace{\frac{\partial l_s^1}{\partial w}}_{<0} + \underbrace{\frac{\partial l^x}{\partial w}}_{<0} G(a_2^x) \right) = \underbrace{l^x (l_s^0 - l_s^1)}_{<0} g(a_1^x).$$

This last condition implies that $\frac{\partial w}{\partial a_1^x} > 0$. Finally, to show that $\frac{\partial \bar{w}}{\partial l^x} = 0$, the proof proceeds similarly to that of lemma 2. ■

Proposition 9

1. *$\tilde{U}(l^x, \lambda)$ achieves a global maximum in $[l_{min}^x, l_{max}^x]$ at some labor threshold $l_{pe}^x \in (l_{min}^x, l_{max}^x)$ characterized by*

$$l_{pe}^x = \sup_{l^x} \tilde{U}(l^x, \lambda).$$

Suppose that $g(\cdot)$ satisfies $g' < 0$, then,

2. *$\tilde{U}^e(a_1^x, \lambda)$ and $\tilde{U}^w(a_1^x, \lambda)$ are strictly concave in a_1^x.*

3. *The equilibrium labor threshold l^x_{pe} under flexible wages is the unique solution to,*

$$\lambda \frac{\partial \tilde{U}^w(l^x_{pe}, \lambda)}{\partial l^x} + (1-\lambda) \frac{\partial \tilde{U}^e(l^x_{pe}, \lambda)}{\partial l^x} = 0 \tag{D.15}$$

Proof: Rewrite equations (6.5) and (6.6) as a function of a^x_1 and differentiate in terms of a^x_1,

$$\frac{\partial \tilde{U}^e}{\partial a^x_1} = \int_{\underline{a}_0}^{a^x_1} \frac{\partial U^e(a, l(a)|x_0)}{\partial a^x_1} \partial G + \frac{\partial U^e(a^x_1, l^x|x_0)}{\partial a^x_1}[G(a^x_2) - G(a^x_1)] + \int_{a^x_2}^{a_M} \frac{\partial U^e(a, l(a)|x_1)}{\partial a^x_1} \partial G + [U^e(a^x_1, l(a^x_2)|x_0) - U^e(a^x_1, l(a^x_2)|x_1)]g(a^x_2), \tag{D.16}$$

$$\frac{\partial \tilde{U}^w}{\partial a^x_1} = \int_{\underline{a}_0}^{a^x_1} \frac{\partial U^e(a, l(a)|x_0)}{\partial a^x_1} \partial G + \frac{\partial U^w(a^x_1, l^x|x_0)}{\partial a^x_1}[G(a^x_2) - G(a^x_1)] + \int_{a^x_2}^{a_M} \frac{\partial U^w(a, l(a)|x_1)}{\partial a^x_1} \partial G + [U^w(a^x_1, l(a^x_2)|x_0) - U^w(a^x_1, l(a^x_2)|x_1)]g(a^x_2). \tag{D.17}$$

Proof of Item 1

Using equations (D.16) and (D.17), the proof proceeds similarly to that of proposition 5.

Proof of Item 2

Differentiation of equations (D.16) and (D.17) gives,

$$\frac{\partial^2 \tilde{U}^e}{\partial a^{x\,2}_1} = -2 \underbrace{\left[\frac{\partial U^e(a^x_2, l(a^x_2)|x_1)}{\partial a^x_1} - \frac{\partial U^e(a^x_2, l(a^x_2)|x_0)}{\partial a^x_1} \right]}_{>0} \cdot \underbrace{\frac{\partial a^x_2}{\partial a^1_x}}_{>0} - \underbrace{\left[U^e(a^x_2, l(a^x_2)|x_1) - U^e(a^x_2, l(a^x_2)|x_0) \right]}_{<0} \underbrace{g'(a^x_2)}_{<0} \underbrace{\frac{\partial a^x_2}{\partial a^1_x}}_{>0},$$

$$\frac{\partial^2 \tilde{U}^w}{\partial a^{x\,2}_1} = -2 \underbrace{\left[\frac{\partial U^w(a^x_2, l(a^x_2)|x_1)}{\partial a^x_1} - \frac{\partial U^w(a^x_2, l(a^x_2)|x_0)}{\partial a^x_1} \right]}_{>0} \cdot \underbrace{\frac{\partial a^x_2}{\partial a^1_x}}_{>0} - \underbrace{\left[U^w(a^x_2, l(a^x_2)|x_1) - U^w(a^x_2, l(a^x_2)|x_0) \right]}_{?} \underbrace{g'(a^x_2)}_{<0} \underbrace{\frac{\partial a^x_2}{\partial a^1_x}}_{>0},$$

where I have used the results from propositions 1 and 2 that $\frac{\partial^2 U^e}{\partial a \partial x} > 0$, $\frac{\partial^2 U^w}{\partial a \partial x} > 0$, and that $\frac{\partial U^e}{\partial x} < 0$. Thus, if $g' < 0$, then $\frac{\partial^2 \tilde{U}^e}{\partial a^{x\,2}_1} < 0$. To show that $\frac{\partial^2 \tilde{U}^w}{\partial a^{x\,2}_1} < 0$ proceed as in the proof of item 2 of proposition 5.

Proof of Item 3

Since both $\tilde{U}^e(a^x_1)$ and $\tilde{U}^w(a^x_1)$ are strictly concave in a^x_1, then $\tilde{U}(a^x_1) = \lambda \tilde{U}^e(a^x_1) + (1-\lambda)\tilde{U}^e(a^x_1)$ is strictly concave. The size threshold that maximizes $\tilde{U}(a^x_1)$, denoted by a^x_{pe}, satisfies,

$$\frac{\partial \tilde{U}(a^x_{pe})}{\partial a^x_1} = 0,$$

$$\Leftrightarrow \frac{\partial \tilde{U}(l^x_{pe})}{\partial l^x} \cdot \underbrace{\frac{\partial l^x}{\partial a^x_1}}_{>0} = 0,$$

where the last condition leads to (D.15). ■

D.5 Asset-based policy: self-reporting

Sections 5.2 and 6.1 have shown that the equilibrium EPL is S-shaped regardless on whether regulations are defined based on assets or labor. Also, the asset-based welfare is larger than the labor-based welfare due to the distortions generated by strategic behavior under a labor-based policy. Why in practice governments don't implement EPL in terms of assets?

In the baseline model of section 3, I have assumed that firms' assets are observable. But in reality firms can decide how many assets to report. Consider an economy where the government can implement a labor policy contingent in assets, but where firms report their assets. In this case, firms may want to under-state their assets in order to operate under a less protective EPL. However, under-reporting involves a cost: since banks restrict credit to less-capitalized agents, under-reporting means they have less access to credit than if they reported truthfully. Thus, under-reporting means: i) more flexible EPL, but at the cost of ii) lower investment.

If effect ii) dominates, then no firm would have incentives to lie about its assets holdings. If that is the case, then an asset-based policy would not create any distortion on welfare and would be preferable over a labor-based policy. Lemma 5 shows that this is not the case. Given some asset threshold above which EPL becomes stricter a^x, there is a range of entrepreneurs with $a \geq a^x$ that claim to have slightly less wealth than a^x. That is, they under-report their size. As a result, they invest less in a firm than if they reported truthfully, but gain from reduced labor costs. As in the case of a labor-based policy, strategic behavior distorts welfare by constraining the extent to which an S-shaped EPL can generate "cross-subsidies" through wages.

Lemma 5 *There exists a critical value $\bar{\epsilon} > 0$ such that agents with $a \in [a^x, a^x + \bar{\epsilon})$ report having slightly less assets than a^x.*

Proof: Consider an agent endowed with wealth $a = a^x + \epsilon$, where $\epsilon > 0$. Thus, if she reports her assets truthfully she invests $k = a^x + \epsilon + d(a^x + \epsilon)$ and hires $l = l(a^x + \epsilon)$ units of labor. The utility she obtains from reporting a is given by

$$U^e(a|x_1) = pf(k,l) + (1-p)\eta k - \bar{w}(x_1)l - (1+\rho)d,$$

Otherwise, if she under-reports her size and says that she owns slightly less than a^x, then her utility is given by

$$U^e(a^x|x_0) = pf(k^x, l^x) + (1-p)\eta k^x - \bar{w}(x_0)l - (1+\rho)d^x,$$

where $k^x = a^x + d(a^x)$ and $l^x = l(a^x)$. Define the following auxiliary function,

$$h(\epsilon) \equiv U^e(a|x_1) - U^e(a^x|x_0) = p[f(k,l) - f(k^x,l^x)] + (1-p)\eta[k - k^x] - \bar{w}(x_1)l + \bar{w}(x_0)l^x - (1+\rho)[d - d^x]. \tag{D.18}$$

First, note that,

$$h(\epsilon)\big|_{\epsilon=0} = \bar{w}(x_0)l^x - \bar{w}(x_1)l < 0,$$

since $\bar{w}(x_0) < \bar{w}(x_1)$ and $l > l^x$. Secondly, differentiate $h(\epsilon)$ in terms of ϵ,

$$\begin{aligned}
\frac{\partial h(\epsilon)}{\partial \epsilon} &= U^e_k(a|x_1)\frac{\partial k}{\partial \epsilon} + U^e_l(a|x_1)\frac{\partial l}{\partial \epsilon} + U^e_d(a|x_1)\frac{\partial d}{\partial \epsilon}, \\
&= \underbrace{[p f_k(k,l) + (1-p)\eta]}_{>0}\left(1 + \frac{\partial d}{\partial \epsilon}\right) + \underbrace{[p f_k(k,l) - (1+r^*)]}_{\geq 0}\frac{\partial d}{\partial \epsilon} > 0,
\end{aligned}$$

where I have used that $\frac{\partial d}{\partial \epsilon} = \frac{\partial d}{\partial a}\frac{\partial a}{\partial \epsilon} > 0$, since $\frac{\partial d}{\partial a} > 0$. Finally, since $h(0) < 0$, $h' > 0$ and h is continuous in ϵ, there is a unique $\bar{\epsilon} > 0$ such that $h(\bar{\epsilon}) = 0$. Thus, any agent with assets $a \in [a^x, a^x + \bar{\epsilon})$ is better off by reporting slightly less assets than a^x. ■

D.6 General regulations

Suppose that regulations are given by some function $\mathcal{P} : [0, a_M] \to [0, 1]$ that maps firms assets into firm's specific strength of regulations, i.e. $\mathcal{P}(a) = \nu(a)$. The government can increase the strength of regulations from ν_0 to $\nu_1 = \nu_0 + \Delta$, with $\Delta > 0$.

Regulations are translated into a payment, $\tau^e(a; w, \rho, \nu, \mathcal{P})$ that must be made by an entrepreneur with assets a who wants to operate a firm, and as a transfer, $\tau^w(l_s; w, \rho, \nu, \mathcal{P})$ to a worker who is supplying l_s units of labor. Note that payments and transfers can depend on assets (a) or labor supplied (l_s), prices (w and ρ), firm's specific regulations ($\nu \equiv \nu(a)$) and regulations applied to other firms ($\mathcal{P}$). To simplify the exposition, suppose that if a firm invest k and hires l units of labor, then output is $f(k, l)$ with certainty. Thus, there is no bankruptcy or a job separation probability.

Thus, the utility of an entrepreneur with assets a who is subject to regulations ν is,

$$U^e(a|\nu) = f(k, l) - wl - (1 + \rho)d - \tau^e(a; w, \rho, \nu, \mathcal{P}) - F. \tag{D.19}$$

The utility of an individual worker who supplies l_s units of labor in a firm under regulations ν is given by

$$u^w(l_s|\nu) = wl_s + \tau^w(l_s; w, \rho, \nu, \mathcal{P}) - \varsigma(l_s). \tag{D.20}$$

The parameter ν measures the strength of regulations faced by an entrepreneur that starts a firm with assets a. In this section, I show how these framework can be used for the study of other size-contingent regulations. These regulations can be divided into two categories: taxes or subsidies to *labor* and *capital* use.

D.6.1 Labor use

Regulations may impose a cost to labor use. In the paper I focused on dismissal regulations. Thus, τ^e was proportional to the labor income owed to workers in a given firm, $w \cdot l(a)$. Additionally, this payment was made only if the worker was fired. Therefore, τ^e was paid with probability s in case of individual dismissal and $(1 - p)$ in case of collective dismissal.

However, $\tau^e(a; w, \rho, \nu)$ can represent more general labor regulations, such as safety standards, working conditions, health insurance, training subsidies, among other employment regulations that are also size-contingent. For instance, in France firms reaching 50 employees must form a committee for hygiene, safety and work conditions, as well as pay higher payroll rates to subsidize training (Gourio and Roys, 2014). These costs can be interpreted as a variable tax on labor use that firms must pay in order to operate. These costs are proportional to the total labor hired by the firm,

$$\tau^e(a; w, \rho, \nu, \mathcal{P}) = \nu \cdot l(a), \tag{D.21}$$

and thus workers matched to that firm receive benefits given by

$$\tau^w(l; w, \rho, \nu, \mathcal{P}) = \nu \cdot l(a). \tag{D.22}$$

In this case, ν can be interpreted as the strength of labor regulations or as a measure of employment' benefits in a given firm.

D.6.2 Capital use

D.6.2.1 Size-restrictions Governments may impose a tax on firms growing too large. For example, Japan and France impose restrictions on the expansion of the retail sector (see Bertrand and Kramarz, 2002, for a discussion of the French case). Under these rules, retail businesses must follow a special procedure to obtain a license for the expansion of existing retail businesses, or for the opening of new stores beyond a size threshold.

In this case, the cost for capital use can be modeled as a tax proportional to total capital invested,

$$\tau^e(a; w, \rho, \nu, \mathcal{P}) = \nu \cdot k(a), \tag{D.23}$$

where ν captures the differences in taxes on capital use across firms with different sizes. Households (workers) receive a lump-sum transfer,

$$\tau^w(l; w, \rho, \nu, \mathcal{P}) = \frac{\int_{\underline{a}_0}^{a_M} \nu k(a) \partial G}{G(\underline{a})}, \tag{D.24}$$

where note that in this case τ^w does not depend on which firm the worker is matched to.

D.6.2.2 Financial subsidies In many countries smaller firms receive credit subsidies. For instance, South Korea provides large financial subsidies for smaller firms (Guner et al., 2008). These policies can be modeled in terms of changed credit costs,

$$\tau^e(a; w, \rho, \nu, , \mathcal{P}) = \nu \cdot \rho d(a). \tag{D.25}$$

Thus, the 'effective' credit cost of a firm with debt $d(a)$ is given by $(1 + \rho(1 + \nu))d(a)$. A credit or interest rate subsidy can be represented by a low (or negative) ν relative to other firms. As before, workers receive a lump-sum transfer,

$$\tau^w(l;w,\rho,v,,\mathcal{P}) = \frac{\int_{\underline{a}_0}^{a_M} v\rho d(a)\partial G}{G(\underline{a})}, \tag{D.26}$$

D.6.2.3 Special tax treatments In many developed and developing countries SMEs enjoy of special tax treatments, such as a reduction of property tax payments or corporate tax rates (e.g US, UK, Belgium, Germany). Additionally, in many countries tax enforcement increases with size (for recent evidence, see Bachas et al., 2019). These types of policies can be interpreted as a tax on firm's assets which varies across firms through v,

$$\tau^e(a;w,\rho,v,\mathcal{P}) = v \cdot a. \tag{D.27}$$

Again, workers receive,

$$\tau^w(l;w,\rho,v,\mathcal{P}) = \frac{\int_{\underline{a}}^{a_M} va\partial G}{G(\underline{a})}. \tag{D.28}$$

E Appendix: Additional Figures

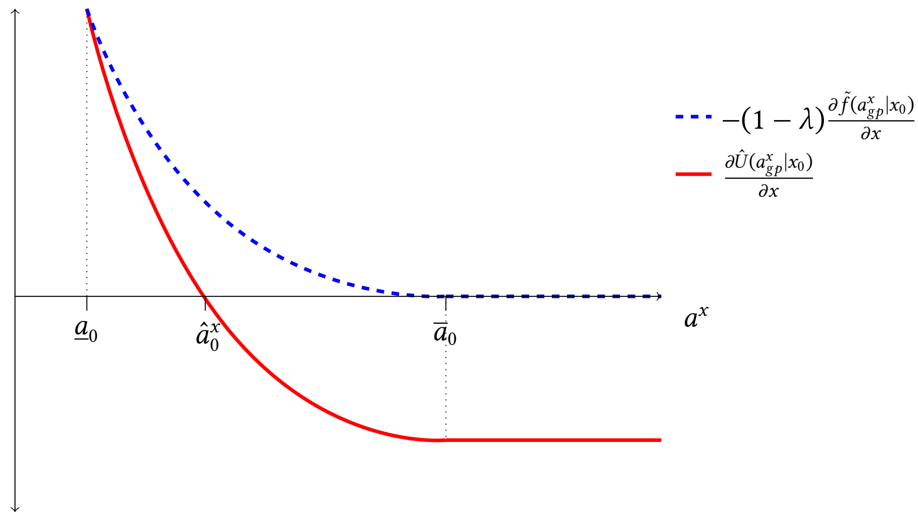

Figure 23: FOC as function of a^x under sticky wage when $\lambda \leq \frac{1}{2+1/(\gamma-2)}$.

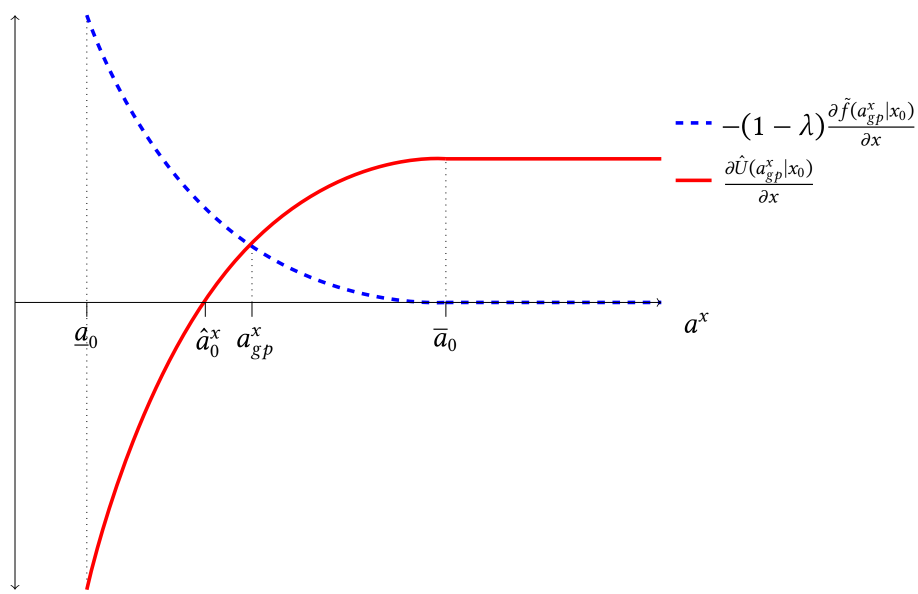

Figure 24: FOC as function of a^x under sticky wage when $\lambda > \frac{1}{2-1/\gamma}$.